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The generation of hierarchic structures via robust 3D topology optimisation

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ABSTRACT

Commonly used building structures often show a hierarchic layout of structural elements. It can be questioned whether such a layout originates from practical considerations, e.g. related to its construction, or that it is (relatively) optimal from a structural point of view. This paper investigates this question by using topology optimisation in an attempt to generate hierarchical structures. As an arbitrarily standard design case, the principle of a traditional timber floor that spans in one direction is used. The optimisation problem is first solved using classical sensitivity and density filtering. This leads indeed to solutions with a hierarchic layout, but they are practically unusable as the floor boarding is absent. A Heaviside projection is therefore considered next, but this does not solve the problem. Finally, a robust approach is followed, and this does result in a design similar to floor boarding supported by timber joists. The robust approach is then followed to study a floor with an opening, two floors that span in two directions, and an eight-level concrete building. It can be concluded that a hierarchic layout of structural elements likely originates from being optimal from a structural point of view. Also clear is that this conclusion cannot be obtained by means of standard topology optimisation based on sensitivity or density filtering (as often found in commercial finite element codes); robust 3D optimisation is required to obtain a usable, constructible (or in the future: 3D printable) structural design, with a crisp black-and-white density distribution.

1. Introduction

Hierarchic layouts of structural elements are commonly found in building structures such as floors, walls and roofs. A hierarchic structure consists of sets of structural elements and arranges a flow of forces hierarchically from the last set on which an external load acts via intermediate sets to the primary set which is connected to a supporting structure. The flow of forces is stepwise being concentrated by each set with the aim to transfer a distributed load to a few specific locations. For example, a floor can be spanned by primary bridging joists, secondary beams, and floor boarding, or a cladding system via primary to tertiary elements, see Fig. 1. Using hierarchic layouts of structural elements could very well originate from practical considerations, e.g. in terms of construction speed and preventing errors. Namely, its aspects of repetition allow for standardized building methods, and errors are spotted more easily in a repetitive system. However, a hierarchic layout of structural elements could also be explained if it would be (relatively) optimal from a structural point of view. The goal of the research presented here is to investigate whether hierarchic structures can be found using topology optimisation, and as such to demonstrate that structural optimality is one of the contributing factors in their design. A second goal of this paper is to demonstrate the applicability of several approaches to topology optimisation in the context of a real structural design problem.

The aim of topology optimisation is to find the optimal distribution of material in a certain design domain. In the context of structural design, minimum compliance problems are often considered. In this case, next to the design domain, the loads and boundary conditions are given, as well as the amount of available material, and the aim is to find the distribution that minimizes the compliance of the structure. In the density-based approach to topology optimisation [1,2], which is one of the most widely used approaches, the design domain is subdivided into a large number of finite elements. A so-called density is assigned to each element. The element densities control the distribution of material: an element with zero density is void, an element with unit density is solid. Intermediate densities (grey elements) are also allowed in

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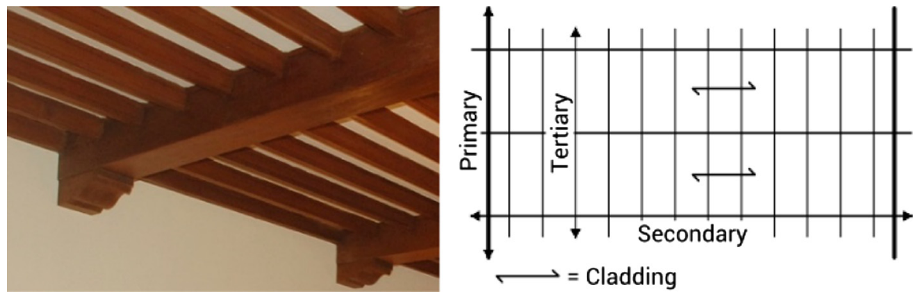

Fig. 1. On the left three levels of hierarchy: bridging joists on corbels, secondary beams, and floor boarding (hidden by plaster), Old Town Hall, Beek, The Netherlands. On the right a schematic view of a cladding system with four levels of hierarchy.

order to obtain a continuous optimisation problem, but they are penalised by a penalisation factor, see Eq. (1) in Section 2. This penalisation factor leaves densities with values 0 and 1 unmodified, however intermediate densities are reduced. This operation is carried out to approach a "black-and-white design", containing only densities with values 0 (white) and 1 (black) at the end of the optimisation. An iterative procedure is followed to find the optimal material distribution: for each iteration, a finite element analysis is performed, the deformation energy per element is determined, and the material is redistributed by moving material from locations where the material is less efficient to locations where the material is more efficient. In order to ensure the existence of solutions and to suppress the occurrence of checkerboard patterns in the optimised design [3–5], filtering techniques are commonly used. An overview paper has been published that first summarizes existing filters and then introduces new morphology based filters, e.g. eroding, dilating, opening, and closing morphology filters, and compares these with existing filters like density filtering, sensitivity filtering, and filters based on a Heaviside projection [6]. Finally, topology optimisation has been used to optimise all sorts of artefacts, e.g. bridges, aircraft wings, chairs and tables, statues, etc. [7]. Other research applies topology optimisation in design process simulation e.g. for automated building spatial and structural design [8]. More specifically related to this paper, plate optimisation via topology optimisation has been studied [9,10], but they used finite elements based on plate theory. These plate elements are 2D, and consequently do not allow for differentiation of the material distribution over the height of the design domain. The influence of different types of plate formulation has been studied as well [11]. In order to reduce computational time, symmetry can be used [12], and useful analytical benchmarks can be found as well [13]. Related to this a derivation of an analytical solution is found in [14]. It can be concluded that a number of techniques exist for topology optimisation, which have been used for a variety of design problems. However, a study on the optimality of hierarchic 3D structures has not yet been published. This will be the contribution of this paper, focussing on a standard (hierarchic) design problem: a commonly used timber floor.

This paper is organised as follows. Sections 2 and 3 address the optimisation of a timber floor structure using standard sensitivity and density filtering, respectively. It is shown that these filters do not lead to a crisp black-and-white solution. In Section 4, a Heaviside projection is added in order to solve this problem, but to no avail. In Section 5, a robust filter is therefore used. This filter has originally been proposed to improve the robustness of the optimised design with respect to geometric imperfections [15], but it has been shown to lead to very crisp black-and-white designs in situations where all other filters fail [16]. Also for the standard problem considered in the present paper (a timber floor), the robust filter performs very well. In Section 6, the optimised floor designs are discussed, and in Section 7, three additional applications are considered: a floor with an opening, two floors that span in two directions, and an eight-level concrete building. Finally, Sections 8 and 9 present a discussion, and conclusions and recommendations respectively.

2. Sensitivity filtering

The focus of this paper is on a standard (hierarchic) design problem: a commonly used timber floor that consists of bridging joists with a cross-section of 38 × 235 mm and a centre to centre distance of 300 mm [17]. The span length equals 5.0 m and the floor boarding has a thickness equal to 18 mm. The design modulus of elasticity is 6000 N/mm^2 and the Poisson's ratio is assumed to be 0.3. The floor is loaded with a uniformly distributed load p_d equal to 3.0 kN/m^2 and is simply supported; see Fig. 2.

The design domain used to formulate the optimisation problem is defined based on the geometry of the timber floor described above. Symmetry is used to reduce computational costs by modelling half the floor joists' span length. An intermediate part of a wider floor is modelled: symmetry conditions are applied at the left and right side, resulting in a model of an infinitely wide floor (see Fig. 3).

In this section, the optimisation problem is solved by means of a sensitivity filtering based approach as described by [1]. The problem is formulated as follows:

$$\min_{\mathbf{x}} c(\mathbf{x}) = \mathbf{u}^{\mathrm{T}}\mathbf{K}\mathbf{u} = \sum_{e=1}^{n} (x_e)^p \mathbf{u}_e^{\mathrm{T}} E_0 \mathbf{K}_e \mathbf{u}_e \quad (1)$$

subject to:

$$\frac{V(\mathbf{x})}{V_0} = f \quad (2)$$

$$0 < x_{\min} \leqslant x_e \leqslant 1 \quad (3)$$

where the displacements $\mathbf{u}$ are found by solving the system of equilibrium equations:

$$\mathbf{K}\mathbf{u} = \mathbf{f} \quad (4)$$

In Eqs. (1)–(4), the objective is the minimisation of compliance c, which is related to the total strain energy over all elements, the latter expressed as a function of the global displacement vector $\mathbf{u}$ and global stiffness matrix $\mathbf{K}$. In the objective function, e is a finite element identifier, n the total number of elements, and x_e is the density of element e, where all densities are combined in a vector $\mathbf{x}$. The variable p is a penalisation factor, $\mathbf{u}_e$ is the displacement vector of an element and $\mathbf{K}_e$ is an element's stiffness matrix (without Young's modulus E_0, which is separately added in the equations, for consequent definitions in subsequent sections). The constraint in Eq. (2) keeps the ratio between the structural volume (being a function