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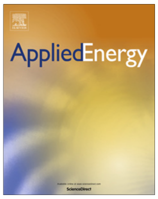

Parameter sensitivity analysis of duplex Stirling coolers

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HIGHLIGHTS

- The performance of duplex Stirling coolers is sensitive to a number of parameters.
- Two methods are presented to make the power piston's displacement constant.
- A new configuration is introduced for duplex Stirling coolers.
- The new duplex Stirling cooler shows similar characteristics as the typical configuration.

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ABSTRACT

Duplex Stirling coolers are external heat-driven regenerative machines that obtain cooling power with high efficiency and reliability. Although the technology is promising, little progress has been reported since they were invented. This paper examines the influence of system parameters on the cooling performance of a duplex Stirling cooler. The study showed that the system was sensitive to a number of parameters, as slight changes in these parameters resulted in the cooling performance deviating significantly from the design. In practice, deviation of system parameters from the design is inevitable. Therefore, actual systems often produce unexpected performance, and this is likely the main obstacle to utilizing such a system in practical applications. To overcome this disadvantage, two methods were proposed to allow the power piston to move with a constant displacement, which may allow for duplex Stirling coolers to become a more widely used cooling system. A new configuration for duplex Stirling coolers was also considered, and it showed similar characteristics as the typical configuration.

1. Introduction

In heat-driven regenerative coolers, high temperature thermal energy is used to generate cyclic pressure fluctuations in heat engines, which are used to produce cooling in refrigerators. The thermal energy can come from any heat source including concentrated solar energy, combustion of fossil fuels or waste material, or the waste thermal streams of industrial plants or power systems. The ability to use thermal energy to produce refrigeration makes these coolers especially attractive where electrical energy is deficient. One potential application is natural gas liquefaction, which is accomplished by burning part of the gas [1]. At present, the highest reported efficiency of heat-driven regenerative natural gas liquefaction has been achieved by burning about 30% to liquefy the rest [2,3].

There are three kinds of heat-driven regenerative coolers: the Vuilleumier cooler, the heat-driven thermoacoustic cooler, and the duplex Stirling cooler. In a Vuilleumier cooler, pressure fluctuations are produced by shuttling gas periodically from an ambient temperature region to a high temperature region by the action of a reciprocating displacer in a hot cylinder. There is another displacer in the cold cylinder to pump heat. The hot and cold displacers are coupled by a kinematic mechanism to maintain proper phasing and synchronous operation. The friction between the displacers and the cylinders makes the Vuilleumier cooler susceptible to failure, which limits its application [4]. In a heat-driven thermoacoustic cooler, the engine and refrigerator are coupled acoustically and the pressure fluctuates spontaneously as long as the heating temperature is maintained [5–7]. There are no moving parts, which results in reliability and simplicity. The working frequency of this cooler is determined by the length of the resonant tube [8]. However, the flow loss in the resonant tube is significant, especially when the pressure ratio is high, so the efficiency of the thermoacoustic cooler is low [9]. In a duplex Stirling cooler, a power piston

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Nomenclature

A_{pis}	piston area (m^2)	i	$\sqrt{-1}$
I	current (A)	Im	imaginary part of a complex number
K_{pis}	spring constant (N/m)	M_{pis}	piston mass (kg)
P_{en}	fluctuating pressure in engine (Pa)	P_{pis}	fluctuanting pressure in front of piston (Pa)
P_{re}	fluctuating pressure in refrigerator (Pa)	Q_c	cooling power (W)
Q_h	heating power (W)	Re	real part of a complex number
R_{elec}	electric resistance (Ω)	R_{mech}	Mechanical resistance (kg/s)
T_c	cooling temperature (K)	T_h	heating temperature (K)
T_r	room temperature (K)	U_{en}	volume flow rate of engine (m^3/s)
U_{pis}	volume flow rate of piston (m^3/s)	U_{re}	volume flow rate of refrigerator (m^3/s)
v	velocity of piston (m/s)	$W_{acou,en}$	acoustic work out of engine (W)
$W_{acou,re}$	acoustic work into refrigerator (W)	$W_{dissi,pis}$	acoustic work dissipated by the power piston (W)
X_{elec}	imaginary of electric impedance	Z_{elec}	electric impedance of the alternator (Ω)
Z_{en}	acoustic impedance of the engine (Pa·s/m)	Z_{mech}	mechanical impedance of piston (N·s/m)
Z_{re}	acoustic impedance of the refrigerator (Pa·s/m)	η_e	exergy efficiency
τ	transduction coefficient (N/A)	ω	angular frequency (rad/s)
$\lvert\ \rvert$	magnitude of a complex number		

is employed to couple a free-piston engine and a free-piston refrigerator. Two displacers are used to adjust the phase [10]. Compared with a heat-driven thermoacoustic cooler, there are no long tubes so the flow loss is significantly reduced. Furthermore, the moving parts are suspended by flexure or gas bearings, which eliminates frictional contact. Therefore a duplex Stirling cooler has the advantages of high efficiency, high reliability, and compactness.

The free-piston Stirling concept was introduced by Beal in 1969 [11]. In recent years, it has drawn worldwide attentions due to the urgent need for renewable energy technologies [12–18]. In the 1980s, several duplex Stirling machines were developed by Sunpower Inc. and while experimental results demonstrated their feasibility, detailed parameters of these machines and quantitative results were not presented [19–21]. With the exception of several analyses [22–25], very little progress regarding duplex Stirling machines has been reported since. Compared with a free-piston engine or cryocooler [17,26], the duplex configuration is more complex, which inevitably increases the difficulty in building such a system. Steady state operation can be achieved only when the acoustic power dissipated by the power piston is equal to the difference between the powers delivered by the engine and absorbed by the refrigerator. This paper will quantify the energy balance and demonstrate how the performance is influenced when system parameters deviate from the designed values. Several methods will be introduced to keep the system working steadily. To find possible ways to avoid parameter sensitivity, a new configuration for the duplex Stirling cooler will be developed. Finally, conclusions will be drawn.

2. Parameters of the system

2.1. Design for the engine and refrigerator

Fig. 1a shows the schematic of a duplex Stirling cooler design, which includes three main components: a free-piston Stirling engine, a free-piston Stirling refrigerator, and a power piston between the engine and refrigerator. The designed working frequency is 50 Hz and the charging pressure is 6 MPa. Sharing the same power piston, the engine and the refrigerator have the same swept volume. To ensure the system can work independently, the engine must output more acoustic power than the refrigerator absorbs since the power piston will dissipate some work. So the refrigerator was first designed independently, then the parameters of the engine is adjusted to satisfy this.

For the refrigerator with the parameters in Table 1, it absorbs an acoustic work $W_{acou,re}$ of 7247 W and produces a cooling powerQ_c of 2043 W at 110 K, corresponding to a coefficient of performance ($Q_c/W_{acou,re}$) of 0.282. Its acoustic impedance Z_{re} is$7.80 \times 10^7 \angle -79.4^\circ$ Pa·s/m. The swept volume of the power piston is 638 cc. Given the diameter and mechanical resistance in Table 1, the piston displacement of 14 mm and dissipated power $W_{dissi,pis}$ of 970 W are obtained. So the engine must output acoustic work$W_{acou,en}$ of 8217 W. After adjusting, the parameters for the engine are then obtained as shown in Table 1. It absorbs 16.1 kW heating power Q_h at 873 K, corresponding to a thermal efficiency ($W_{acou,en}/Q_h$) of 50.9%. Its acoustic impedance Z_{en} is$7.41 \times 10^7 \angle 77.3^\circ$ Pa·s/m. If the overall exergy efficiency is defined as

$$\eta_e = \frac{Q_c T_h (T_r - T_c)}{Q_h (T_h - T_r) T_c}, \tag{1}$$

the exergy efficiency for this system is 33.3%, which indicates that consuming about 11% of the gas is sufficient to liquefy the remainder; this is comparable to conventional large-scale liquefiers based on Joule-Thomson or Brayton cycles [27,28].

Above calculation for the refrigerator and engine is carried out separately in two separate numerical programmes based on SAGE [29].Once the engine and refrigerator are couple together by the power piston, an energy balance will be built among $W_{acou,en}$, $W_{acou,re}$ and $W_{dissi,pis}$. In a practical system, many parameters such as the mechanical resistance of the power piston, charging pressure, and heating temperatures, et al. may deviate from the designed values, which will destroy the designed energy balance. The system may change its working parameters to reach another balance instead of the designed. $W_{dissi,pis}$ is very small compared $W_{acou,en}$ and $W_{acou,re}$. Little change of $W_{acou,en} - W_{acou,re}$ may result great change of piston displacement since $W_{dissi,pis}$ is a function of piston displacement. So, how the system will respond to the parameter change must be further investigated.

2.2. Design for the total duplex Stirling cooler

With the Newton's law for the power piston, there is [30],

$$P_{en} - P_{re} = \frac{Z_{mech}}{A_{pis}^2} U_{pis} \tag{2}$$

If the phase of U_{pis} is set to zero, the phasor diagram of the pressures and volume flow rate can be shown as in Fig. 1b. The