Solar flux density calculation for a heliostat with an elliptical Gaussian distribution source

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Abstract

The calculation of solar flux density is a key work for the design and optimization of solar tower system. Because of the great amount of calculation, the source distribution is often regarded as a radial distribution, which is not consistent with the reality. This paper presents a new method to calculate the flux density distribution by a focusing heliostat with an elliptical Gaussian distribution source. The two-dimensional convolution integration is proposed and converted into a one-dimensional integration. We use the Gauss-Legendre integration method to reduce the amount of calculation and accelerate the speed of integration. This method can be used to calculate the solar flux at image and receiver plane by most of the heliostat. It needs only 0.1% time of the ray tracing method for calculating the efficiency of the heliostat. It can be applied for design optimization of the solar heliostat field which is superior to the present methods in both accuracy and computation requirements.

1. Introduction

Solar tower system uses a lots of heliostats to reflect the solar energy to a central receiver, which is one of the potential solar energy technology [1,2]. We need to calculate the solar flux at the receiver to predict the temperature of the receiver [3], and prevent the flux density being more than the receiver’s tolerance limit [4–6]. We should also calculate the solar energy intercepted by the receiver [7] from the energy flux density distribution to evaluate system performance, and optimize the system design. Therefore, the test and calculation of the solar flux density distribution produced by a single heliostat, is one of the critical steps for developing solar tower systems [8]. To simulate them, many codes have been developed since the mid-seventies by different groups [9]. They divide into two main types according to their calculation method [10]: convolution-based and ray-tracing.

Ray tracing is an ordinary method for heliostat study [11] or other concentrated solar collector [12], which tracks the solar ray path reflected by the reflector in the system and the receiving surface position they reach, the solar intensity distribution at any surface can be calculated with a high degree of flexibility to adapt to various situations, and programming is simple for obtaining a reliable result. For example, recently, it was applied to study the solar flux formed by a non-imaging focusing heliostat [13]. The disadvantage is that it needs more computing resources to obtain highly accurate and consistent calculation results than the convolution-based methods, not suitable for system optimization [14] although the Monte Carlo Ray-Trace (MCRT) method may reduce computation resources [15].

Convolution-based integration method requires less computing resources, and thus subjects to researchers’ attention. As the solar ray reflected by a point of the heliostats from different solar position will reach to different positions of the receiver surface, plus dispersion caused by the optical error, we need to calculate the solar intensity which reached the specific position from all the reflection points on the heliostats by integration to obtain the solar energy density. We can first calculate the solar flux at the image...
Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>B</td>
<td>distribution function of reflected sun brightness (W/m²/rad)</td>
</tr>
<tr>
<td>C</td>
<td>C_R = I_0</td>
</tr>
<tr>
<td>C_R</td>
<td>the concentration ratio of heliostats</td>
</tr>
<tr>
<td>DNI</td>
<td>the direct normal irradiance (W/m²)</td>
</tr>
<tr>
<td>F</td>
<td>solar flux (W/m²)</td>
</tr>
<tr>
<td>l</td>
<td>the normalization constant for solar brightness distribution</td>
</tr>
<tr>
<td>I_0</td>
<td>the intensity of reflected light (W/m²)</td>
</tr>
<tr>
<td>I</td>
<td>the distance from a point at image plane to the reflection point</td>
</tr>
<tr>
<td>M</td>
<td>function of the heliostat image at image plane</td>
</tr>
<tr>
<td>O</td>
<td>the central point of the image of the heliostat</td>
</tr>
<tr>
<td>P</td>
<td>a point at the heliostat</td>
</tr>
<tr>
<td>Q</td>
<td>a point at the image plane for flux calculation</td>
</tr>
<tr>
<td>r</td>
<td>the radial coordinate variable</td>
</tr>
<tr>
<td>r_0</td>
<td>the distance between the point of O and Q</td>
</tr>
<tr>
<td>\Gamma_R, \Gamma_B</td>
<td>the integration range at r direction</td>
</tr>
<tr>
<td>R</td>
<td>the image radius of the round heliostats at image plane</td>
</tr>
<tr>
<td>R_0</td>
<td>the radius of the round heliostats (m)</td>
</tr>
<tr>
<td>S</td>
<td>the scope of integration = the circle image of heliostats in the image plane</td>
</tr>
<tr>
<td>x_0, y_0, x, y</td>
<td>coordinate</td>
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Greek symbol

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<tr>
<td>\gamma</td>
<td>incident angle of solar ray to the heliostat</td>
</tr>
<tr>
<td>\theta</td>
<td>radial angular displacement or angular displacement in transverse direction (rad)</td>
</tr>
<tr>
<td>\varphi_0</td>
<td>limits of the integration at angle direction (rad)</td>
</tr>
<tr>
<td>\rho</td>
<td>the reflectivity of the heliostat</td>
</tr>
<tr>
<td>\sigma_{tx}</td>
<td>the standard deviation of the Gaussian distribution at x direction</td>
</tr>
<tr>
<td>\sigma_{ty}</td>
<td>the standard deviation of the Gaussian distribution at y direction</td>
</tr>
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Typical applications for the convolution methods are field layout optimizations and system performance estimations. The inaccuracy of the convolution methods are often ignored as main errors for performance prediction of solar tower system do not come from the optical model but from the other component such as the storage, and turbine [9]. However, recent developments, such as layout optimization of the field, aiming strategy optimization to avoid too much more flux density and improve the flux distribution [5], require high accuracies and fast computations [24,25].

The reflected solar ray is closer to an elliptical Gaussian distribution which is known for many years [26,27]. Collado [8] evaluated it with the experimental data to show that the maximum relative error is more than 10% for interceptor calculation of a rectangular heliostat with a circular Gaussian distribution source. The main reasons include, the first, the main optical error, the slope error of the optical system is elliptical Gaussian distribution [28], not circular Gaussian distribution, which leads to an elliptical Gaussian distribution of reflected solar brightness. The second, even the slope error distribution is circular Gaussian distribution, when the incident angle is more than 0, which is the ordinary case in solar tower system, the reflected solar brightness is still more closer to an elliptic Gaussian distribution [29,30].

So we should apply the elliptical Gaussian distribution to describe the reflected solar brightness distribution for solar flux density distribution calculation. In Helios code [27], an algorithm based on the two dimensional fast Fourier transform is applied to solve the more general convolution integration, including the elliptical Gaussian distribution for various heliostats. But computationally intensive, it consumes a lot of computer resources, the actual code is rarely used especially for optimization [9].

In this paper, we present a new numerical method to calculate energy flux density distribution by a focusing heliostat with the elliptical Gaussian distribution sources for improving the calculation accuracy and speed. The application of the methods to the efficiency calculation and design optimization is also analyzed and compared with the other methods.

2. Model

2.1. Model theory

We started from the reflected solar intensity distribution to calculate the solar flux formed by a heliostat. We first analyze the