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Invariant-based formulation of a triangular finite element for geometrically nonlinear thermal analysis of composite shells

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ABSTRACT

The paper describes an invariant-based formulation of a triangular finite element for geometrically nonlinear analysis of shear flexible composite shells subjected to thermal loads. Transverse shear deformation is taken into account using the first order shear deformation theory. The focus is on the representation of the strain energy of the shell in terms of invariant quantities which depend on the components of the strain tensor and elastic constants of the material. Based on the invariant expression for the strain energy density, algorithmic relations are derived for computing the stiffness matrix of the shell finite element. The finite element formulation is used to study stability of equilibrium configurations in the region of large thermal displacements. A positive definite second variation of the total energy is used as a sufficient criterion for stability of equilibrium configurations. A series of numerical examples are given to estimate performance of the finite element in solving nonlinear problems of composite plates and shells under uniform temperature rise. Solution of some classical problems of laminated plates and shells shows that there exist equilibrium configurations not previously reported in the literature.

1. Introduction

Composite structures made of two or more different materials combined together offer significant advantages over traditional structural materials. Owing to their high stiffness-to-weight and strength-to-weight ratios, composite materials are increasingly used in aerospace, automotive, shipbuilding industries, etc. Therefore much effort has been concentrated on the development of effective methods for predicting deformations and stresses in composite structures subjected to various loading conditions. Excellent review of the computational models of laminated and sandwich plates and shells can be found in [1–3]. Most theoretical studies have dealt with laminated plates and shells under mechanical loads, whereas less attention has been given to thermal loading conditions. When subjected to heating, thin-walled structural members exhibit nonlinear response and can buckle due to compressive stresses that develop under restrained thermal expansion. Temperature rise is an important loading factor to consider in the design of high-speed aerospace vehicles subjected to aerodynamic heating and chemical plants operating at elevated temperature.

Approximate analytical solutions governing thermal response of composite structures are limited to simple geometries and typically obtainable for linear problems only (see, e.g. [4–6]). To develop more general methods applicable to practical situations including nonlinear effects, more attention has been focused on finite-element formulations. Here we list some significant contributions in this area.

Thangaratnam et al. [7,8] used the Semiloof shell element in the buckling analysis of thin cylindrical and conical laminated shells subjected to uniform temperature rise. Chen and Chen [9] studied thermal buckling behavior of composite laminated plates using a rectangular four-node finite element with 48 degrees of freedom (DOF). In a recent paper, Ounis et al. [10] developed a four-node element with 32 DOF which is a combination of linear isoparametric membrane element and Hermitian element for plate bending problems. These finite-element formulations are based on the classical laminated plate theory (CLPT), which is valid only for thin laminates.

Refined finite-element solutions of the buckling problems of thermally loaded plates that take into account transverse shear deformation were obtained by Chandrashekhara [11] and Prabhu and Dhanaraj [12]. They used nine-node Lagrangian isoparametric elements based on the first-order shear deformation theory (FOSDT). Their numerical results indicate that the transverse shear deformation has a significant effect on the critical temperature of moderately thick laminates. Using the FOSDT, Dawe and Ge [13] developed the finite strip method for predicting critical buckling

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temperature of rectangular composite plates. For a rectangular plate divided into strips of finite width, they used spline functions to approximate displacement fields along the strips and standard Lagrange shape functions to approximate those across the strips. Kabir et al. [14] extended formulation of a three-node element with full integration scheme to solve thermal buckling problems of thin and moderately thick composite plates. Shiau et al. [15] proposed an 18 DOF triangular element for buckling analysis of symmetrically laminated plates.

The studies mentioned above are restricted to linear buckling analysis where prebuckling deformations are ignored. However, this approach leads to significant errors if prebuckling displacements are large and affect stiffness of the structure. In this case, full geometrically nonlinear analysis should be performed. One of the earliest studies on large thermal deflections of laminated panels was carried out by Huang and Tauchert [16], who used a C^0-continuity quadrilateral shell element based on the FOSDT. Ganapathi and Touratier [17] proposed an eight-node quadrilateral element for postbuckling analysis of laminated plates undergoing moderately large rotations. Using the FOSDT, Barut et al. [18] developed a co-rotational formulation of a curved triangular finite element for predicting nonlinear response of laminated shells under temperature field that varies across the shell surface and through the thickness. Nonlinear thermoelastic deformations of closed cylindrical and conical shells of oval cross section were studied by Patel et al. [19] who used a C^0-continuous eight-node quadrilateral finite element. Based on the results of nonlinear analysis, they concluded that the linear eigenvalue buckling solution leads to a significant error in determining critical temperature. Geometrically nonlinear analysis of shallow and deep shells under hygrothermal environment was performed by Kundu et al. [20]. They employed a nine-node isoparametric doubly-curved element with 45 DOF. Sabik and Kreja [21] discussed the important issue of determining postcritical deformations that refer to solution branches crossing the fundamental deformation path. In a recent paper, Alijani et al. [22] investigated large thermal deformations and snapping behavior of closed cylindrical shells. The temperature rise versus deflection curves were obtained by semi-analytical finite-element formulation based on the FOSDT.

A more accurate analysis of thick composite structures can be carried out using higher-order shear deformation theories (HOSDT), zig-zag theories (ZZT) and layerwise theories (LWT). They could also be represented in a unified form as discussed in a very large body of literature (see, e.g. [23–26]). Finite-element formulations based on the HOSDT for thermal buckling problems of thick plates and shells were proposed in [27–31]. It should be noted, however, that the finite-element implementation of HOSDT requires much more unknown variables compared to the CLPT and FOSDT and leads to dramatic increase in the computational work especially in the nonlinear analysis of shell structures.

An analysis of the literature on thermally loaded composite structures shows that most finite-element formulations deal with the linear buckling problems of laminated plates and shells. Publications on geometrically nonlinear analysis of composite shells under temperature rise are sparse and restricted (with rare exceptions) to investigation of the primary (or fundamental) deformation path. To the authors' knowledge, stability of equilibrium states of thermally deformed shells has received little attention. It is well known that, for nonlinear deformable systems, multiple equilibrium states can occur under the same level of load. From the practical point of view it is important to distinguish between stable and unstable equilibrium configurations of shells in order to infer which configurations and stress distributions can occur in real practical situations.

The aim of the present study is to develop a computationally effective finite element for nonlinear stability analysis of composite thin-walled shells subjected to thermal loads. The core part of the paper is concerned with invariant-based formulation of a curved triangular finite element. In [32–34], triangular finite elements of isotropic shells were developed using an expression for the strain energy in terms of invariants of the strain tensor. The idea was to give a simple and concise finite-element formulation for geometrically nonlinear analysis of two-dimensional structures. In [35], an attempt was made to formulate a triangular element of laminated anisotropic shells using a similar approach. It was found that the strain energy can be written as a function of so-called combined invariants which depend not only on the strain components but also on the stress components. As a result, some computational effort was needed to perform stress-to-strain conversion when deriving the element stiffness matrix. In the present study, the strain energy of anisotropic shell is represented in terms of combined invariants which depend on the strain components and elastic properties of the material. This approach allows one to avoid the additional stress-to-strain conversion in the derivation of the stiffness matrix of the triangular finite element.

The paper is organized as follows. Section 2 describes coordinate transformations which relate Cartesian components of a second rank tensor to its three normal components determined in three independent directions on the plane. Section 3 contains discussion of combined invariant of two tensors. Section 4 focuses on a method for expressing the strain energy density of thermally loaded anisotropic shell based on the FOSDT in terms of combined invariants which depend on the components of the strain tensor and elastic constants of the material. Sections 5 and 6 deal with the formulation of a shell triangular element based on the invariant expression for the strain energy. Section 7 describes a solution procedure for determining equilibrium configurations. Finally, Section 8 presents numerical results obtained by the finite element proposed.

2. Transformation of tensor components

We consider a symmetric second-order tensor u_{mn} ($m, n = 1, 2$) whose independent components in Cartesian coordinates ξ_1 and ξ_1 are denoted by u_{11}, u_{22}, and u_{12}. For further derivation, it is convenient to write the tensor in the vector form

$$\boldsymbol{u} = \{u_{11},\ u_{22},\ u_{12}\}^T. \tag{2.1}$$

The tensor can also be represented by its three normal components u_i ($i = 1, 2, 3$) determined in three independent directions on the plane. When dealing with triangular domains, it is reasonable to use three normal components of the tensor related to three coordinate lines q_i parallel to the triangle edges (see Fig. 1).

Following terminology of Argyris (see, e.g. [36–38]), we call these three normal components the natural components of the tensor. Thus, in addition to (2.1), any symmetric two-dimensional tensor u_{mn} can be represented by the vector containing three natural components

$$\boldsymbol{u} = \{u_1,\ u_2,\ u_3\}^T. \tag{2.2}$$

Let us give explicit relations between Cartesian and natural components of a two-dimensional tensor. To this end, we consider a triangle and denote its vertices by i, j, and k. In what follows, we employ summation over dummy indices unless otherwise specified. To describe the material properties, we use Cartesian coordinates ξ_1 and ξ_2. The natural components determined in the directions q_i are given by (no summation over i)

$$u_i = \alpha_{mni} u_{mn}, \tag{2.3}$$