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Three dimensional coupled thermoelasticity solution of sandwich plate with FGM core under thermal shock

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ABSTRACT

Based on the Lord–Shulman formulation, time dependent response of sandwich plate with functionally graded material (FGM) core is performed by using generalized coupled thermoelasticity. Governing partial differential thermoelasticity equations are reduced to ordinary differential equations by applying Fourier series state space technique and then are solved analytically via Laplace transform. Solutions are then converted to time domain by using inverse Laplace transform. Validation of the present approach is assessed by comparing the numerical results with the available results in literature. Finally influence of time constant, applied temperature, mid radius to thickness ratio and time history on transient thermoelastic behavior of sandwich plate are studied.

1. Introduction

Sandwich plates are widely used in modern engineering applications, especially in mechanical industries, nuclear reactors and aeronautical industry due to its lightweight, high stiffness, high structural efficiency and strength. Matching the material properties in the layers interface causes to overcome to interfacial delamination as well as to shear cracking the core. Recently, many researchers have studied mechanical properties and behavior of FG sandwich structures. Based on three-dimensional theory of elasticity, Li et al. [1] studied free vibration of simply supported and clamped edges sandwich plates with volume fraction distribution of constituents according to simple power law. Kant et al. [2] used two-point boundary value problem (BVP) governed by a set of linear first-order ordinary differential equations (ODEs) through the thickness of a laminate to carry out static analysis of composite and sandwich laminates. Brischetto [3] investigated static behavior of sandwich plates with material properties according to the Legendre polynomials by using classical and mixed advanced models. Tu et al. [4] studied bending and vibration of laminated and sandwich composite plates by using finite element method and considering parabolic distribution of the transverse shear strains as well as rotary inertia. Based on high-order shear deformation theory (HSDT), Abdelaziz et al. [5] investigated static behavior of FG sandwich plate. Natarajan and Manickam [6] employed QUAD-8 shear flexible element developed based on higher order structural theory to analysis of bending and free vibration of sandwich FGM plate. Static analysis of FG sandwich plates was carried out by Neves et al. [7] using hyperbolic sine term for the in-plane displacements and a quadratic function of thickness coordinate for transverse displacement. Khalili et al. [8] studied free vibration of sandwich plates with FG face sheets for various thermal environments using Hamilton's principle and assumption of the material properties of FG facing sheets to be third-order function of temperature. Xiang et al. [9] studied free vibration of sandwich plate with FG face sheets and homogeneous core by using meshless global collocation method based on two-dimensional shear deformation theory. Loja et al. [10] employed B-spline finite strip element models based on various shear deformation theories to investigate static and free vibration of FG sandwich plate with assumption of material properties to obey Mori-Tanaka formulation. Based on a higher-order shear deformation theory, Neves et al. [11] analyzed static, free vibration and buckling behaviors of simply supported isotropic and sandwich FG plates using meshless technique. Tounsi et al. [12] carried out thermoelastic analysis of simply support FG sandwich plates using refined trigonometric shear deformation theory. Based on three dimensional theory of elasticity, static and free vibration analysis of two types of sandwich rectangular plates was carried out by Alibeigloo and Alizadeh [13] using state space differential quadrature method.

Mentioned research works are about static or vibration behavior of sandwich plate subjected to thermal and /or mechanical load. Transient response of plate structures subjected to thermal or mechanical shock loads have been reported in literature. Three-

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Nomenclature

a, b, h	length, width and thickness of sandwich plate along the x-, y- and z- directions, respectively
h_m, h_c, h_f	thickness of metal, ceramic and FGM layers, respectively
E_m, α_m, k_m, ρ_m, E_c, α_c, k_c, ρ_c	Young's modulus, thermal expansion coefficient, thermal conductivity , material density of metal and ceramic, respectively
n, m	half wave number along longitudinal and width direction, respectively
q	heat flux
R	internal heat generation
u, v, w	displacement components in the x-, y- and z- direction, respectively
Y_o, ρ_o, K_o, c_o, α_o, T_o	characteristic values of the elastic modulus, density, thermal conductivity, specific heat, thermal expansion coefficient and temperature
τ_o	thermal relaxation time
$\sigma_i(i = x, y, z)$	normal stresses
$\tau_{xy}, \tau_{xz}, \tau_{yz}$	shear stresses
$\varepsilon_i(i = x, y, z)$	normal strains
$\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$	shear strains
ν	Poisson ratio

dimensional coupled thermoelasticity analysis of a functionally graded rectangular plate was carried out by Zhou et al. [14] using Lord-Shulman theory and state space technique approach. Wang et al. [15] used Clausius inequality and generalized theory of thermoelasticity to study thermoelastic of thin plate under with variable material properties and subjected to thermal shock. Based on different theory of thermoelasticity, Zenkour [16] presented three dimensional temperature, displacement and stresses distribution of plated subjected to thermal shock. Based on Lord and Shulman theory, Wang et al. [17] presented transient thermoelastic response of a FG thin plate analytically using Laplace transform technique. Based on Euler beam theory and Vonkarman-type nonlinear strain displacement, Zhong et al. [18] investigated nonlinear thermal response of FGM beams subjected to thermal shock and rested on elastic foundation using DQM, Neumark and Newton–Raphson method. Based on layer wise higher-order theory, Pandey and Pradyumna [19] presented thermoelastic response of FGM sandwich beam under thermal shock using finite element formulation and Crank-Nicolson method. Based on first order shear deformation plate theory, Based on First order shear deformation plate theory, Jafarinezhad and Eslami [20] analyzed coupled thermoelasticity of FGM annular plate subjected to transverse thermal shock using Laplace transform for time domain and Galerkin finite element method. According to the above mentioned reviewing the literature, it was found that three dimensional transient coupled thermoelastic response of simply supported sandwich rectangular plate under thermal shock has not yet been reported. In the present work governing first order differential equations are derived by applying Fourier series state space technique to the constitutive relations, equation of motion and generalized coupled thermoelasticity heat conduction equation based on the Lord–Shulman theory. Then governing state space equations in time domain are converted to Laplace domain which can be solved analytically. Solutions are then converted to time domain by using inverse Laplace transformation to obtain stress displacement and temperature gradient in time domain.

2. Formulation of basic equations

Consider a simply supported sandwich plate with FGM core layer which is bonded to metal and ceramic facing sheets at its bottom and top surfaces, respectively (Fig. 1). The sandwich plate with initially at the ambient temperature, T_0 is suddenly subjected to thermal shock at the top surface. Thermoelastic material constant in FGM core are assumed to vary along the thickness direction according to the following exponential law

$$E = E_m e^{\beta_1(z-h_m)},\quad \alpha = \alpha_m e^{\beta_2(z-h_m)},\quad K = K_m e^{\beta_3(z-h_m)}\quad \rho = \rho_m e^{\beta_4(z-h_m)} \tag{1}$$

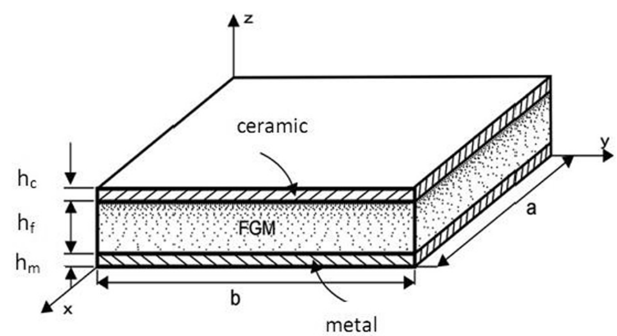

Fig. 1. Geometry of sandwich rectangular plate.

where $\beta_1 = \frac{1}{h_f}\ln\frac{E_c}{E_m}$, $\beta_2 = \frac{1}{h_f}\ln\frac{\alpha_c}{\alpha_m}$, $\beta_3 = \frac{1}{h_f}\ln\frac{K_c}{K_m}$, $\beta_4 = \frac{1}{h_f}\ln\frac{\rho_c}{\rho_m}$ and E_m, α_m, k_m, ρ_m, E_c, α_c, k_c, ρ_c are Young's modulus, thermal expansion coefficient, thermal conductivity coefficient and material density at the bottom and top surface of the FGM core layer.

In the absence of body forces, differential equations of motion in Cartesian coordinate system for FGM layer are

$$\sigma_{x,x} + \tau_{xy,y} + \tau_{xz,z} = \rho\frac{\partial^2 u}{\partial t^2}$$

$$\tau_{xy,x} + \sigma_{y,y} + \tau_{yz,z} = \rho\frac{\partial^2 v}{\partial t^2}$$

$$\tau_{xz,x} + \tau_{yz,y} + \sigma_{z,z} = \rho\frac{\partial^2 w}{\partial t^2} \tag{2}$$

Thermo elastic constitutive relations in term of displacements are

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)u_{,x} + \nu v_{,y} + \nu w_{,z}\right] - \frac{\alpha E T}{1-2\nu}$$

$$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)}\left[\nu u_{,x} + (1-\nu)v_{,y} + \nu w_{,z}\right] - \frac{\alpha E T}{1-2\nu}$$

$$\sigma_z = \frac{E}{(1+\nu)(1-2\nu)}\left[\nu u_{,x} + v_{,y} + (1-\nu)w_{,z}\right] - \frac{\alpha E T}{1-2\nu}$$

$$\tau_{xy} = \frac{E}{2(1+\nu)}(u_{,y} + v_{,x}) \qquad \tau_{xz} = \frac{E}{2(1+\nu)}(u_{,z} + w_{,x})$$

$$\tau_{yz} = \frac{E}{2(1+\nu)}(v_{,z} + w_{,y}) \tag{3}$$

The energy balance equation takes the form