



## Review

# A review of continuum mechanics models for size-dependent analysis of beams and plates



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## ABSTRACT

This paper presents a comprehensive review on the development of higher-order continuum models for capturing size effects in small-scale structures. The review mainly focus on the size-dependent beam, plate and shell models developed based on the nonlocal elasticity theory, modified couple stress theory and strain gradient theory due to their common use in predicting the global behaviour of small-scale structures. In each higher-order continuum theory, various size-dependent models based on the classical theory, first-order shear deformation theory and higher-order shear deformation theory were reviewed and discussed. In addition, the development of finite element solutions for size-dependent analysis of beams and plates was also highlighted. Finally a summary and recommendations for future research are presented. It is hoped that this review paper will provide current knowledge on the development of higher-order continuum models and inspire further applications of these models in predicting the behaviour of micro- and nano-structures.

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## 1. Introduction

Small-scale structural elements such as beams, plates and shells are commonly used as components in micro- and nano-electromechanical systems (MEMS and NEMS), sensors, actuators and atomic force microscopes. In these applications, size effects were experimentally observed in mechanical properties [1–5]. These size effects can be captured using either molecular dynamics (MD) simulations or higher-order continuum mechanics. Although the MD method can provide accurate predictions, it is too computationally expensive. Therefore, higher-order continuum mechanics approach was widely used in the modelling of small-scale structures.

The development of higher-order continuum theories can be traced back to the earliest work of Piola on the 19th century as demonstrated in [6–7] and the work of Cosserat and Cosserat [8] in 1909. However, until 1960s, the Cosserat brothers' idea was received considerable attention from researchers, and a large number of higher-order continuum theories have been developed. In general, these theories can be categorized into three different classes namely the strain gradient family, microcontinuum and nonlocal elasticity theories. The strain gradient family is composed of the couple stress theory, the first and second strain gradient theories, the modified couple stress theory and the modified strain gradient theory. In the strain gradient family, both strains and gradient of strains are considered in the strain energy, and thus the size effect is accounted for using material length scale parameters. In the couple stress theory initiated by Toupin [9], Mindlin and Tiersten [10] and Koiter [11], only the gradient of rotation vector is considered in the strain energy, and thus only two material length scale parameters are required. The modified couple stress theory was proposed by Yang et al. [12] based on modifying the couple stress theory. By introducing an equilibrium condition of moments of couples to enforce the couple stress tensor to be symmetric, the number of material length scale parameters of the

modified couple stress theory is reduced from two to one. The first strain gradient theory initiated by Mindlin [13] considers only the first gradient of strains. One year later, Mindlin [14] derived the second strain gradient theory which is considered as the most general form of strain gradient theory accounting for both the first and second gradients of strains. Lam et al. [15] proposed the modified strain gradient theory with only three material length scale parameters by modifying Mindlin's theory by using a similar approach of Yang et al. [12]. The microcontinuum theory was developed by Eringen [16–18] consisting of micropolar, microstretch and micromorphic (3M) theories. Micropolar theory which is actually initiated by Cosserat brothers [8] is the simplest one among 3M theories, whilst micromorphic theory is the most general one among 3M theories. In 3M theories, each particle can rotate and deform independently regardless of the motion of the centroid of the particle. More details about the 3M theories as well as their applications can be found in [19–25]. The nonlocal elasticity theory was originally proposed by Kroner [26] and improved by Eringen [27–28] and Eringen and Edelen [29]. In this theory, the stress at a reference point in a continuum depends on the strains at all points of the body, and thus the size effect is captured by means of constitutive equations using a nonlocal parameter. Nonlocal elasticity theory was initially formulated in an integral form and later reformulated by Eringen [30] in a differential form by considering a specific kernel function. Compared to the integral model, the differential one is widely used for nanostructures due to its simplicity. In addition, another class of higher-order theory which is called nonlocal strain gradient theory has been recently proposed based on a combination of the nonlocal elasticity theory and the strain gradient theory. The interested reader can refer to [31–33] for more details on this theory.

Size-dependent models have been widely used for predicting the global behaviour of beam- and plate-like nanostructures such as carbon nanotubes (CNTs) and graphene sheets. CNTs were discovered by Iijima [34] by rolling graphene sheets. Based on

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