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Finite element estimates of viscoelastic stiffness of short glass fiber reinforced composites

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ABSTRACT

An unstructured mesh Galerkin finite element method is used to obtain estimates of the viscoelastic moduli of short glass fiber reinforced polymer composites. Periodic Monte Carlo models with 125 identical inclusions are studied. Both spheroidal and spherocylindrical inclusions are considered. The estimates are compared against predictions of the dilute approximation, Mori-Tanaka (MT) and self-consistent (SC) models. It is shown that at small fiber fractions, the dilute approximation (Eshelby) model is exact. However, for the axial stiffness the dilute regime is limited to fiber volume loadings of a few tens of a percent while typical short glass fiber polymer composites have fiber loadings from 10 to 20 percent. It is found that in this concentrated regime, both MT and SC models give excellent predictions for all but the axial stiffness modulus. To assess the feasibility of reliable stiffness and vibration damping design of composite structures from short fiber reinforced polymers, Monte Carlo models with various fiber orientation distribution (FOD) states are studied. It is shown that the quick Voigt (constant strain) orientation averaging procedure gives excellent viscoelastic stiffness predictions provided that the finite element estimates are used for the required moduli of the basis FOD state with fully aligned fibers.

1. Introduction

Short glass and carbon fiber reinforced polymers are widely used in various automotive, construction, marine, building and other applications [1–4]. Compared to unfilled polymers, such composites typically have enhanced specific stiffness and strength so their use allows for the design of lighter engineering structures with advanced mechanical characteristics. Injection molding and extrusion compounding are commonly used to convert commodity thermoplastic polymers such as nylon, polypropylene and polystyrene [5,6]. And appealingly, one can also use these traditional processing routes to manufacture composite structures from short fiber reinforced polymers [1–4].

Upon processing of short fiber reinforced polymers, spatially non-uniform fiber orientation distributions (FOD) are commonly developed across the final composite structure. As a result, the local elastic stiffness moduli become both position dependent and anisotropic so for optimum structural design, one would naturally like to employ the elastic moduli predicted as a function of local FOD. This is commonly achieved in two separate steps [7–10]. In the first step, one considers a unidirectional FOD with perfectly aligned fibers and predicts its transversely isotropic elastic stiffness moduli using either an analytical micromechanics model or a finite element calculation [8–12]. In the second step, one evaluates the elastic moduli of all the different multidirectional FOD developed in the molded structure in course of processing using a quick Voigt orientation averaging procedure involving the second and fourth order fiber orientation tensors and the elastic moduli of the basis unidirectional FOD obtained in the first step [7,13]. We shall review this two-step procedure below in Sections 3 and 6.

Solid polymers exhibit viscoelastic mechanical responses in which they store the deformation energy as an elastic solid and dissipate it as a viscous fluid [5,14–17]. The vibration damping properties of the manufactured parts can impair both the service life of the composite structures and also their noise emission behavior so it would be desirable to extend the traditional design methods developed for the static elastic responses of short fiber reinforced composite to their viscoelastic responses.

In steady state harmonic oscillations the tensor of viscoelastic moduli, the stiffness tensor, is complex, $\mathbf{C} = \mathbf{C}' + i\mathbf{C}''$, where $\mathbf{C}'$ and $\mathbf{C}''$ are the real (storage) and imaginary (loss) parts, respectively. To predict the vibration damping behavior, both $\mathbf{C}'$ and $\mathbf{C}''$ are required. Using the elastic-viscoelastic correspondence principle, one can convert static elastic solutions of analytical models to steady state harmonic solutions simply by replacing the con-

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stituents' static elastic moduli by the corresponding viscoelastic ones [18]. However, to my best knowledge, for short fiber reinforced polymers such viscoelastic predictions have not yet been validated so it is unclear if they are sufficiently accurate to allow for reliable design.

In this work, following on from the recent developments in time domain unstructured mesh finite element micromechanics calculations [19,20], results are presented, for the first time to my knowledge, on controlled accuracy estimates of the effective viscoelastic stiffness moduli of short fiber composites with both spheroidal and spherocylindrical inclusions. These estimates are used to assess the suitability of classical dilute approximation, Mori-Tanaka and self-consistent models for reliable stiffness and vibration damping design of advanced composite structures from short glass fiber reinforced polymer composites.

2. Time domain finite element calculations

2.1. Periodic Monte Carlo snapshots

Starting from a regular simple cubic configuration with 125 non-overlapping identical spheres, we performed Monte Carlo (MC) runs by moving sequentially the spheres and accepting all attempted configurations without overlapping spheres and rejecting all those with overlapping spheres, see Fig. 1.

Periodic boundary conditions were imposed during the MC runs. For spherical inclusions, we used cubic boxes with unit edges. The sphere diameters were set to achieve the desirable sphere volume fraction υ_f. MC runs of 10^7 attempted MC moves per sphere were conducted. The amplitude of the moves was adjusted to yield a target acceptance ratio of 0.5. For sphere fractions $\upsilon_f < 0.45$, the box edges were kept unchanged during the MC runs. At larger sphere fractions, the MC runs were started from a regular simple cubic array at $\upsilon_f = 0.45$ and then, after sufficiently long randomization, variable shape MC runs were performed to compress the models to the target volume fractions [19]. It has already been shown that such periodic MC models are representative so they allow one to obtain accurate estimates of the effective viscoelastic stiffness of random microstructure composites [19–22].

Fig. 2 shows a Monte Carlo model with 125 identical spheroids aligned along the x-axis.

The equation for a spheroidal particle centered at the coordinate origin is given by

$$\frac{x^2}{a^2} + \frac{y^2 + z^2}{b^2} = 1, \tag{1}$$

where a is the distance from center to pole along the symmetry axis x and b is the equatorial radius of the spheroid. The spheroid aspect ratio is then defined as $\alpha = a/b$.

The box edges of the MC models were set to have the same aspect ratio as the inclusions, with equal unit edges along the

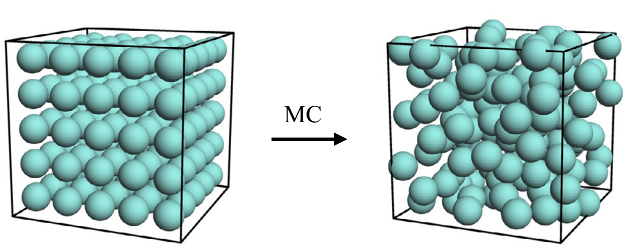

Fig. 1. Generation of periodic random Monte Carlo models with non-overlapping identical spherical inclusions dispersed at a sphere volume fraction of $\upsilon_f = 0.3$.

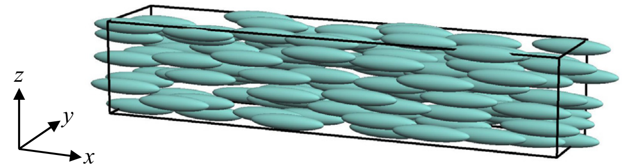

Fig. 2. Periodic random MC model with 125 aligned non-overlapping identical prolate spheroids of aspect ratio $\alpha = 5$ randomly dispersed at a volume fraction of $\upsilon_f = 0.3$.

y- and z-axis. In this work, we studied spheroidal inclusions with α from 1 (spheres) to 1000 assuming υ_f up to 0.55. The desired volume fraction υ_f was achieved by assigning a suitable value of the equatorial radius b. The studied ranges of α and υ_f included those characteristic of commonly used industrial chopped glass fiber reinforced polymer composites, with those having fibers of aspect ratio from $20 < \alpha < 50$ usually termed short fiber composites and those with $\alpha > 100$ termed long fiber composites.

2.2. Periodic unstructured meshes

An in-house Delaunay mesh generator was used to create periodic unstructured morphology-adaptive quality meshes of tetrahedral elements for random MC models with spherical inclusions. The mesh generation procedure was already described elsewhere [11,19]. The largest MC model studied was the one with 125 inclusions randomly dispersed at $\upsilon_f = 0.55$. Its unstructured mesh had ca. $7 \cdot 10^5$ tetrahedrons [19–23].

To generate a periodic mesh for a random MC model with N spheroidal inclusions of aspect ratio α dispersed at volume fraction υ_f, we used the mesh of an MC model with N spherical inclusions dispersed at the same υ_f and applied to all its nodal coordinates an affine transformation in which the nodal x-coordinates were scaled by factor α while the y- and z-coordinates were left unchanged. The same affine transformation was also applied to the edges of the periodic box. This simple scaling procedure allows us to achieve accurate microstructural representation of the MC models with even highly elongated spheroids already with a relatively small number of tetrahedrons, though admittedly highly distorted with their aspect ratios being of the same order of magnitude as the aspect ratios of the scaled boxes so a rather high, third order polynomial interpolation in space was then required to obtain accurate viscoelastic effective stiffness estimates, see Section 2.6 below.

2.3. Effective viscoelastic moduli

We assume harmonic oscillations at angular frequency ω and study linear viscoelastic composites consisting of homogeneous phases with constitutive relations of the form

$$\boldsymbol{\sigma} = \mathbf{C}'(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_0) + (\mathbf{C}''/\omega)(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}_0) \tag{2}$$

where $\mathbf{C} = \mathbf{C}' + i\mathbf{C}''$ is the fourth order tensor of complex viscoelastic moduli, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma}$ are instantaneous second order strain and stress tensors at a given position, respectively, and an overbar denotes a time derivative [20]. Tensor $\mathbf{C}$ is assumed to be time independent and uniform inside the inclusion and matrix phases. In this work, we shall use a direct notation, in which the symbols have direct interpretation as tensors. The initial strain $\boldsymbol{\varepsilon}_0$ is harmonic:

$$\boldsymbol{\varepsilon}_0(t) = \mathbf{E}_0 \sin(\omega t) \tag{3}$$

where t is time and ω the angular frequency. The strain amplitude tensor $\mathbf{E}_0$ is uniform and it defines the imposed mode of deformation [20]. Without loss of generality, in calculations we assumed