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Semi-analytic model to evaluate non-regularized stresses causing unfolding failure in composites

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ABSTRACT

A novel semi-analytic model is developed to determine interlaminar stresses in laminates including highly curved parts, for obtaining a rapid, accurate and reliable design tool for evaluating the reserve factor of a certain aeronautic composite component prone to unfolding failure. The model assumes plane conditions and is based, firstly, in an approximation of the 2D displacement components by using a series of functional products of separate variables and, secondly, in the use of higher-order moments of the stress components, which allow an enriched beam model to be constructed. The solution is obtained as the sum of products of a set of known functions, written in terms of the eigenvalues and eigenvectors of the matrix governing the model, and a set of integration constants. These constants can be determined with the boundary conditions, in isolated parts, and the continuity conditions, in laminates containing parts with different curvatures. In comparison with an existing analytical method, the present model has the advantage of offering an extremely accurate solution at the zones in which the analytical method may yield errors of even a 100%. In comparison with a detailed finite elements model, the present model obtains a similar accuracy in much lower computational times.

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1. Introduction

In the design of aircraft components, each component is subjected to different load cases and may suffer different failures. In order to select the optimum stacking sequence to fulfil all design requirements, simplified analytical solutions are employed to estimate the reserve factor of each configuration, since a detailed FEM analysis of all possible situations, involving thousands of cases, is not feasible for time and resources limitations.

At present, the unfolding failure is one of the cases in which preliminary designs (based on simplified analytical solutions) are very conservative and more precise analytical solutions are required.

The unfolding failure is a failure mechanism mainly consisting in the sudden appearance of delamination in curved parts. It is typically observed in L-shaped components, T-shaped components, joggles, corrugated laminates and other laminates containing highly curved parts loaded with opening bending moments, where the interlaminar stresses reach significant values [1,2].

Classical calculation methods for composite laminates (see for instance [3], Chapter 4) consider only the intralaminar stresses and neglect the interlaminar stresses, because they are very small with respect to the former in most composite applications. However, when the laminate is curved with $R \sim t$ (where R is the mean radius of the laminate and t is its thickness) interlaminar stresses reach significant values, and the classical calculation methods are not able to appropriately describe the failure of the laminate [4].

The unfolding failure has been widely studied in L-shaped and T-shaped components. These thin wall sections are typically modelled with a bi-dimensional approximation and using beam theory. The curved parts of the section are isolated and considered as curved beams of constant radius.

Initially, interlaminar normal and shear stresses in composite curved laminates were calculated replacing the laminate with an equivalent homogeneous material [5–9] and using Lekhnitskii's equations to determine stresses in curve anisotropic beams (see [10], Chapter 3). Kedward et al. [1] presented a very simple and widely used equation to determine the maximum of the through-thickness component of the stresses calculated using Lekhnitskii's equations. Following Cui et al. [11], the maximum of

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Lekhnitskii's shear stress is typically approximated by the expression used in straight beams.

A general methodology for calculating all the components of the stresses in a layered curved beam, under a bi-dimensional assumption, was given by Ko and Jackson [12]. This solution, which is an extension of the Lekhnitskii's equations from the homogeneous material to the laminated case, is typically employed nowadays in the evaluation of the stresses in composite laminates with curved parts.

Since the solution presented by Ko and Jackson in [12] is difficult to implement, simpler and less accurate solutions have been developed. Some simpler models use the Kirchoff hypothesis over the displacements, which allows an approximated solution of the problem to be obtained [13,14]. This hypothesis neglects the shear strain and stress. More advanced models considering these components [15,16] show that the shear effect is not relevant in the solution. Guedes and Sá [17] avoided the use of the Kirchoff hypothesis by using Fourier series to approximate the displacement field.

All these models [12–17] take into account the non-homogeneity of the material due to the stacking sequence of the laminate, considering the effect of the curvature in both the stiffness parameters and the strain–displacements relations. As a consequence, logarithmic expressions are obtained to evaluate the components of the typical composite stiffness matrix *A-B-D*. The main effect observed when these logarithmic expressions introduced by the curvature are employed is a coupling ($B \neq 0$) between axial and bending forces and strains, which appears even when the laminate is symmetrical. Further simplification in the stress solution can be obtained using the thin laminate hypothesis ($t \ll R$), which implies a calculation of the stiffness parameters similar to the classic flat laminate theory [18]. However, curved laminates suffering unfolding failure have a small R/t ratio (typically $2 < R/t < 4$) and the hypothesis of thin laminate is not valid.

Nevertheless, all the aforementioned analytical methods may calculate only regularized stresses in the curved beam, as they suppose it isolated. As a consequence, their stress solutions do not take into account the so-called edge-effects. That is, stresses do not fulfil neither the boundary conditions at beam ends nor the continuity conditions at the joint of beams with different curvatures. For this reason, when the maximum of the regularized interlaminar stresses is obtained in the vicinity of the aforementioned non-accurate zones, finite elements models have to be employed to determine interlaminar stresses. Comparison of regularized analytical and numerical solutions shows that existing regularized analytical solutions result very conservative in cases when non-regularized stresses are significant, such as non-uniform external pressures being applied to the outer and inner radius of the curved beam, at the joint of beams with different curvature or between beams with different material properties, etc. These effects and the high importance of them are commented by Most et al. in [19].

The most illustrative case of the non-regularized effects is the L-shaped beam, where the section can be modelled as a curved beam joined to two straight beams. When the section is loaded by a pure bending moment, regularized interlaminar stresses are null in the straight beams while they have a not null distribution in the curved beam. As a consequence, a discontinuity is observed in the regularized stresses at the joint between the straight beams with the curved beam. Therefore, a non-regularized stress distribution appears in the vicinity of the joints to ensure a continuous transition from the regularized stresses in the straight beam to the regularized stresses in the curved beam.

As a previous step to calculate the solution of the non-regularized stress distribution in curved beams, the authors presented an alternative procedure to obtain the regularized solution for the stresses in a curved beam [20]. This procedure was based on a layerwise beam theory approximation which has the advantage of considering external pressures applied to the outer and inner sides of the curved laminate. In absence of external pressures, this model has been validated by comparison with the regularized solution and in the presence of external pressures by comparison with the solution given by finite elements models [21]. The model divides every ply of the curved laminate in several fictitious laminas that are approached by the Timoshenko beam theory [22,23].

In this paper, moving one step further from the Classical Beam Theory (CBT) applied in [20], a novel model is developed by approximating the displacements using a series of functional products of separate variables, suppressing the Timoshenko hypothesis, and using higher-order moments of the stress components. This novel semi-analytical model allows the non-regularized effects to be taken into account, by imposing at the joints between beams with different curvature not only the continuity of the CBT displacements, forces and moments, but also those higher-order terms. The accuracy of the model depends on the order of the approximation of the series expansion of the displacements, so that when a higher order is chosen the model converges to the exact solution, not analytically known.

The idea of employing a series expansion of the displacements and beam theory to determine interlaminar stresses in curved laminates was also applied by Matsunaga [24,25], who developed a higher-order model based on a series expansion of the displacements using monomial functions. The main drawback of this method is that every term of the series expansion is again approximated by means of a second series expansion in the axial direction of the beam using trigonometric functions. In the present model this second series expansion is not needed, since appropriate auxiliary functions are employed in the series expansion of the through-thickness displacements as well as in the definition of the higher-order moments, which enables the problem to be reduced to a linear system of differential equations whose solution is again reduced to the determination of the eigenvalues and eigenvectors of the system matrix. The removal of this secondary series expansion reduces the computational times and increases the accuracy for the same model order.

In Section 2 the problem under study, a 2D section of a laminate of constant thickness constituted by a chain of constant-curvature curved beams and straight beams, is properly defined. Equations governing the solution of a typical constant-curvature curved beam are detailed in Section 3, and the simplifications associated to the solution of a typical straight beam are described in Section 4. In these sections, the model is reduced to a linear system of differential equations whose solution is obtained in Section 5, as the sum of the products of a set of known functions and a set of integration constants. The equations employed to determine the integration constants from the continuity requirements in the joint of two consecutive beams are briefly described in Section 6. Finally, the solution of the model is validated in Section 7 by comparison with regularized analytical solutions and finite elements solutions for a typical case.

2. Theoretical basis

The main objective of the present paper is the calculation of the non-regularized solution for the stresses in a thin-walled section of a composite component, as those seen in Fig. 1, by means of a bi-dimensional approximation. This 2D section is considered to have a constant thickness, $\bar{t}$, and is typically composed by a chain of straight beams and constant-curvature curved beams, as seen in Fig. 2, with a total number of N_b beams. The section is loaded in its plane under end axial, shear and bending loads. Variable curvature sections can also be considered, approximating them by a chain of constant-curvature beams such as those previously mentioned.

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