Composite Structures 171 (2017) 198–216

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Three-dimensional exact electric-elastic analysis of a multilayered two-dimensional decagonal quasicrystal plate subjected to patch loading

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ARTICLE INFO

Article history:
Received 27 December 2016
Revised 23 January 2017
Accepted 1 February 2017
Available online 10 February 2017

Keywords:
Electric-elastic
Quasicrystal
Patch loading
Exact solution

ABSTRACT

An exact electric-elastic analysis of a multilayered two-dimensional decagonal quasicrystal plate subjected to patch loading with simply supported boundary conditions is presented. The pseudo-Stroh formalism and propagator matrix method are used to obtain the exact three-dimensional mechanical behaviors of the plate. By expressing the patch loading in the form of a double Fourier series expansion an exact closed-form solution with a concise and elegant expression is deduced. Three different kinds of patch surface loadings are applied to the surface of the plate and the response of the plate is investigated. Comprehensive numerical results are shown for a sandwich plate subjected to the three patch loadings with two different stacking sequences. The results show that the stacking sequences, patch loading areas, and patch loading types have a great influence on the stress, displacement and electric components of the plate. Also, different coupling constants between the phonon and phason fields will influence the physical quantities. The useful features observed from numerical results can be used in the design of composite laminates made of two-dimensional piezoelectric quasicrystals. The numerical results can also serve as a reference in convergence studies of other numerical methods and for verification of existing or future plate theories.

1. Introduction

Since the first observation of quasicrystals (QCs) by Shechtman in the early 1980s [1], the structural, electronic, magnetic, thermal and mechanical properties of the material have been investigated extensively. Among approximately 200 individual QCs observed to date, two-dimensional (2D) QCs with fine thermal stability play an important role in this classification of matter [2]. Based on the symmetry breaking principle of Landau, the elastic energy theory of QCs has been formulated [3]. In this elastic theory, there are two lower frequency excitations: phonon and phason. Phonons are related to translations of atoms (standard elasticity), while phasons are related to rearrangements of atomic configurations along the quasiperiodic direction. The generalized linear elasticity of QCs established by Ding [4] provides us with a fundamental theory to describe the elastic behavior of QCs. Based on the elasticity of QCs, expressions of other physical properties of QCs, such as thermal expansion and piezoelectricity tensors, have been obtained [5,6].

Attributing to their properties such as corrosion resistivity, low thermal conductivity, low coefficients of friction, low porosity, high hardness and high wear resistance, QCs have been increasingly applied as thin films and coatings [7] and gained considerable interest in a wide range of study fields, such as dislocations, and defects in infinite spaces and beams or plates or shells [8–10]. The piezoelectric properties are often considered in the works [11–16], so as for defects [17,18]. Since plates are of vital importance in structural design, many analytical solutions for plates have been obtained especially for laminates [19–21]. However, phonon-phason coupling, anisotropy, and nonsymmetry intrinsic in quasicrystalline materials present many obstacles to researchers. For 1D QC plates, many works have been conducted to study the static and dynamic response of the plates, such as static [22] and free vibration response [23] of a multilayered QC plate, the dynamic response of a multilayered QC plate subjected to a patch loading [24]. For 2D QC plates, the complexity of the basic equations of elasticity increases considerably compared to 1D QC which

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http://dx.doi.org/10.1016/j.compstruct.2017.02.036
0263-8223/

limits most of studies on 2D QCs to the defect problems in infinite spaces [25,26]. Although the exact solution for 2D QC laminates has been deduced by Yang [27], no exact closed-form solution for a multilayered plate subjected to patch loading with piezoelectricity has been reported in the literature.

In this paper, we derive an exact electric-elastic solution of a multilayered simply-supported plate made of 2D QCs subjected to patch surface loading. The patch loading is expressed in the form of a double Fourier series expansion. The powerful pseudo-Stroh formalisms [28] are extended to 2D piezoelectric QCs to obtain the general solution for each homogeneous QC layer. The propagator matrix method [28] is then introduced to treat the corresponding multilayered case. Three different kinds of surface patch loadings, which include a transverse shear force, a normal force and an electric potential, are considered in this investigation. Furthermore, a multilayered plate containing both 2D QC layers and crystal layers is investigated. As numerical illustrations, three examples of a multilayered plate subjected to different surface patch loadings are discussed.

2. Basic equations

A 2D QC is a three-dimensional body where its atomic arrangement is quasi-periodic in a plane and periodic along the direction normal to the plane. Referring to a rectangular Cartesian coordinate system (x_1, x_2, x_3), x_1 and x_2 is set as the quasi-periodic directions and x_3 as the periodic direction. Then, phason displacements w_m ($m = 1, 2$) exist in addition to phonon displacements u_i ($i, j = 1, 2, 3$).

The general equations for 2D QCs are given by [6,11]

$$\varepsilon_{ij} = (\partial_j u_i + \partial_i u_j)/2, \quad w_{mj} = \partial_j w_m, \quad E_j = -\partial_j \phi, \tag{1}$$

$$\partial_j \sigma_{ij} = 0, \quad \partial_j H_{mj} = 0, \quad \partial_j D_j = 0, \tag{2}$$

where $\partial_j = \partial/\partial x_j$ and repeating indices imply summation; σ_{ij} and H_{mj} denote the phonon and phason stresses, respectively; ε_{mj} and w_{mj} are the phonon and phason displacements, respectively; D_j, E_j and ϕ represent the electric displacements, electric fields and electric potential, respectively.

For 2D decagonal QCs with the point group $\overline{10}m2$ in Lame 14, the linear constitutive equations of 2D QCs with piezoelectricity can be expressed by the following form [2,3,6]:

$$\begin{aligned}
\sigma_{11} &= C_{11}\varepsilon_{11} + C_{12}\varepsilon_{22} + C_{13}\varepsilon_{33} + R(w_{11} + w_{22}) - d_{31}^{(1)}E_3,\\
\sigma_{22} &= C_{12}\varepsilon_{11} + C_{11}\varepsilon_{22} + C_{13}\varepsilon_{33} - R(w_{11} + w_{22}) - d_{31}^{(1)}E_3,\\
\sigma_{33} &= C_{13}\varepsilon_{11} + C_{13}\varepsilon_{22} + C_{33}\varepsilon_{33} - d_{33}^{(1)}E_3,\\
\sigma_{23} &= \sigma_{32} = 2C_{44}\varepsilon_{23} - d_{15}^{(1)}E_2,\\
\sigma_{31} &= \sigma_{13} = 2C_{44}\varepsilon_{13} - d_{15}^{(1)}E_1,\\
\sigma_{12} &= \sigma_{21} = 2C_{66}\varepsilon_{12} - Rw_{12} + Rw_{21},\\
H_{11} &= R(\varepsilon_{11} - \varepsilon_{22}) + K_1 w_{11} + K_2 w_{22} - d_{112}^{(2)}E_2,\\
H_{22} &= R(\varepsilon_{11} - \varepsilon_{22}) + K_1 w_{22} + K_2 w_{11} + d_{112}^{(2)}E_2,\\
H_{23} &= K_4 w_{23},\\
H_{12} &= -2R\varepsilon_{12} + K_1 w_{12} - K_2 w_{21} - d_{112}^{(2)}E_1,\\
H_{13} &= K_4 w_{13},\\
H_{21} &= 2R\varepsilon_{12} - K_2 w_{12} + K_1 w_{21} - d_{112}^{(2)}E_1,\\
D_1 &= 2d_{15}^{(1)}\varepsilon_{13} + d_{112}^{(2)}(w_{12} + w_{21}) + \kappa_{11}E_1,\\
D_2 &= 2d_{15}^{(1)}\varepsilon_{23} + d_{112}^{(2)}(w_{11} - w_{22}) + \kappa_{22}E_2,\\
D_3 &= d_{31}^{(1)}(\varepsilon_{11} + \varepsilon_{22}) + d_{33}^{(1)}\varepsilon_{33} + \kappa_{33}E_3,
\end{aligned} \tag{3}$$

where C_{ij}, C_{44} and C_{66} are the elastic constants in phonon field; K_1, K_2 and K_4 represent the elastic constants in phason field; R is the coupling constant between the phonon and phason fields; $d_{15}^{(1)}$, $d_{31}^{(1)}$ and $d_{33}^{(1)}$ are the piezoelectric constants in phonon field; $d_{112}^{(2)}$ is the piezoelectric constant in phason field; κ_{11}, κ_{22} and κ_{33} are the permittivity constants.

3. Problem description and general solution for a layered 2D piezoelectric QC plate

Consider a multilayered 2D piezoelectric decagonal QC plate as shown in Fig. 1 with horizontal dimensions $x \times y = L_x \times L_y$ and a total thickness $z = H$ in a rectangular Cartesian coordinate system (x, y, z) with its four sides being simply supported. Although the orientation of the coordinate system can induce different physical fields [27], in this study the case that the global and local coordinate systems have the relation $(x, y, z) = (x_1, x_2, x_3)$ is considered. Accordingly, the periodic direction of the 2D QC is the z-direction or the thickness direction of the plate. Let j denote the j-th layer of the multilayered plate, h_j is defined as the thickness of layer j. Then, the lower and upper interfaces of layer j are defined as z_j and z_{j+1} respectively. It follows that, for an N-layered plate with total thickness H, $z_1 = 0$ and $z_{N+1} = H$. Along the interfaces of the layers, the displacements and z-direction traction stresses are assumed to be continuous, *i.e.*

$$\begin{cases}
(u_i)_j = (u_i)_{j+1}, \quad (w_m)_j = (w_m)_{j+1}, \quad \phi_j = \phi_{j+1},\\
(\sigma_{iz})_j = (\sigma_{iz})_{j+1}, \quad (H_{mz})_j = (H_{mz})_{j+1}, \qquad \text{at the interface of layer } j \text{ and } j+1.\\
(D_z)_j = (D_z)_{j+1}.
\end{cases} \tag{4}$$

The simply supported displacement boundary conditions for the 2D decagonal piezoelectric plate are as follows:

$$\begin{aligned}
x = 0 \text{ and } L_x: \ u_y = u_z = w_y = \phi = 0,\\
y = 0 \text{ and } L_y: \ u_x = u_z = w_x = \phi = 0.
\end{aligned} \tag{5}$$

The solution of the displacement vector of the homogenous 2D QC piezoelectric plate is assumed to take the following form:

$$\mathbf{u} = \begin{Bmatrix} u_x \\ u_y \\ u_z \\ w_x \\ w_y \\ \phi \end{Bmatrix} = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ w_1 \\ w_2 \\ \phi \end{Bmatrix} = \sum_{p,q} e^{sz} \begin{Bmatrix} a_1 \cos px \sin qy \\ a_2 \sin px \cos qy \\ a_3 \sin px \sin qy \\ a_4 \cos px \sin qy \\ a_5 \sin px \cos qy \\ a_6 \sin px \sin qy \end{Bmatrix}, \tag{6}$$

where

$$p = m\pi/L_x, \quad q = n\pi/L_y, \tag{7}$$

with m and n being two positive integers, and s being the eigenvalue to be determined, and a_1, a_2, a_3, a_4, a_5 and a_6 being the components of the corresponding eigenvector to be determined. It can be seen that the displacement vector satisfies the simply supported displacement boundary conditions.

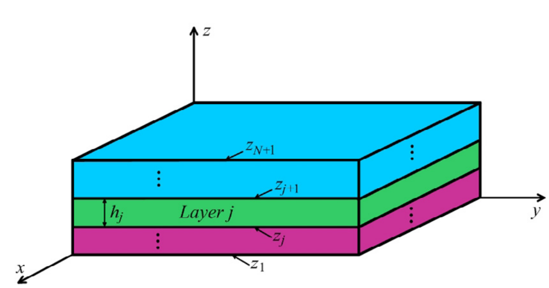

Fig. 1. A multilayered 2D QC plate.