Composite Structures 171 (2017) 277–285

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

A new simple shear deformation plate theory

Huu-Tai Thai [a,*], Trung-Kien Nguyen [b], Thuc P. Vo [c,d,*], Tuan Ngo [e]

[a] School of Engineering and Mathematical Sciences, La Trobe University, Bundoora, VIC 3086, Australia
[b] Faculty of Civil Engineering, Ho Chi Minh City University of Technology and Education, 1 Vo Van Ngan Street, Thu Duc District, Ho Chi Minh City, Viet Nam
[c] Duy Tan University, Da Nang, Viet Nam
[d] Faculty of Engineering and Environment, Northumbria University, Newcastle upon Tyne NE1 8ST, UK
[e] Department of Infrastructure Engineering, The University of Melbourne, Parkville, VIC 3010, Australia

ARTICLE INFO

Article history:
Received 10 November 2016
Revised 11 January 2017
Accepted 8 March 2017
Available online 10 March 2017

Keywords:
Shear deformation theory
Isotropic plate
Analytical solution

ABSTRACT

This paper proposes a new simple shear deformation theory for isotropic plates. The present theory involves one unknown and one governing equation as in the classical plate theory, but it is capable of accurately capturing shear deformation effects. The displacement field of the present theory was based on a two variable refined plate theory in which the transverse displacement is partitioned into the bending and shear parts. Based on the equilibrium equations of three-dimensional (3D) elasticity theory, the relationship between the bending and shear parts was established. Therefore, the number of unknowns of the present theory was reduced from two to one. Closed-form solutions were presented for both Navier- and Levy-type plates. Numerical results indicate that the obtained predictions are comparable with those generated by ABAQUS and available results predicted by 3D elasticity theory, first-order and third-order shear deformation theories.

1. Introduction

Plates and shells are common structural elements in civil engineering structures such as buildings, bridges, tunnels, retaining walls and other infrastructure. In general, the behaviour of plate and shell structures can be predicted using either 2D plate theories or 3D elasticity theory. The classical plate theory (CPT) is the simplest plate theory developed by Love [1] based on the assumptions proposed by Kirchhoff [2]. However, this theory is only applicable for thin plates in which the shear deformation effects are negligibly small. For thick plates, the CPT underestimates deflections and overestimates buckling loads as well as natural frequencies because of neglecting these effects.

A large number of shear deformation theories have been proposed to take into account the shear deformation effects. One of the earliest shear deformation theories was proposed by Reissner [3] and Mindlin [4]. It should be noted that Mindlin's theory was based on an assumption of a linear displacement variation across the plate thickness. It was therefore referred to as the first-order shear deformation theory (FSDT). This assumption leads to constant transverse shear strains and transverse shear stresses across the thickness. A shear correction factor is therefore needed to account for the discrepancy between the constant shear stresses and the parabolic distribution of shear stresses in the 3D elasticity theory. On the other hand, Reissner's theory was based on the assumptions of a linear variation of bending stresses and a parabolic distribution of transverse shear stresses across the thickness. These assumptions lead to a displacement field which is not necessarily linear across the thickness, and the shear correction factor is not required as in the case of Mindlin's theory. Higher-order shear deformation theories (HSDTs) were proposed to eliminate the use of the shear correction factor in the FSDT, and to obtain a better prediction of the responses of very thick plates. The HSDT is developed based on a higher-order displacement variation across the plate thickness using either polynomial functions (e.g. the third-order shear deformation theory (TSDT) of Reddy [5]) or non-polynomial functions (e.g. the sinusoidal theory of Touratier [6], hyperbolic theory of Soldatos [7], exponential theory of Karama et al. [8] and among others). Several typical shear deformation theories developed from 2010 for composite structures can be found in Refs. [9–24]. A comprehensive review of plate theories was reported by Ghugal and Shimpi [25] for isotropic and laminated plates and Thai and Kim [26] for functionally graded plates.

* Corresponding authors at: School of Engineering and Mathematical Sciences, La Trobe University, Bundoora, VIC 3086, Australia (H.T. Thai); Department of Mechanical and Construction Engineering, Northumbria University, Newcastle upon Tyne NE1 8ST, UK (T.P. Vo).

E-mail addresses: tai.thai@latrobe.edu.au (H.-T. Thai), thuc.vo@northumbria.ac.uk (T.P. Vo).

http://dx.doi.org/10.1016/j.compstruct.2017.03.027
0263-8223/

Although the existing HSDTs provide a better prediction compared with the CPT, they are much more complicated and computationally expensive than the CPT because of introducing additional dependent unknowns into the theory. Therefore, this paper aims to propose a simple HSDT which contains the same number of unknowns and governing equations of motion as in the case of the CPT. The present theory was based on the refined plate theory (RPT) of Shimpi [27] and 3D elasticity theory. Analytical solutions of the present theory were also presented. The obtained predictions were then compared with the available results predicted by the FSDT, TSDT and 3D elasticity theory as well as those generated by ABAQUS for validation.

2. Kinematics

The displacement field of the present theory was derived based on the displacement field of the RPT [27] and the equilibrium equations of 3D elasticity theory. According to Shimpi [27], the displacement field of the RPT is given as follows:

$$u(x,y,z,t) = -z\frac{\partial w_b}{\partial x} - z\left[\frac{5}{3}\left(\frac{z}{h}\right)^2 - \frac{1}{4}\right]\frac{\partial w_s}{\partial x} \tag{1a}$$

$$v(x,y,z,t) = -z\frac{\partial w_b}{\partial y} - z\left[\frac{5}{3}\left(\frac{z}{h}\right)^2 - \frac{1}{4}\right]\frac{\partial w_s}{\partial y} \tag{1b}$$

$$w(x,y,z,t) = w_b(x,y,t) + w_s(x,y,t) \tag{1c}$$

where (u, v, w) are the total displacement along the coordinates (x, y, z); w_b and w_s are the bending and shear components of transverse displacement w, respectively; and h is the plate thickness. The equilibrium equations of 3D elasticity theory in the absence of body forces are written as:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = \rho\ddot{u} \tag{2a}$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{y}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = \rho\ddot{v} \tag{2b}$$

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{z}}{\partial z} = \rho\ddot{w} \tag{2c}$$

where the dot-superscript convention indicates differentiation with respect to time t; σ_i are the stress components of the stress tensor; and ρ is the density. Substituting Eq. (1) into Eq. (2), the equilibrium equations are rewritten as:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = -\rho z\frac{\partial \ddot{w}_b}{\partial x} - \rho z\left[\frac{5}{3}\left(\frac{z}{h}\right)^2 - \frac{1}{4}\right]\frac{\partial \ddot{w}_s}{\partial x} \tag{3a}$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{y}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = -\rho z\frac{\partial \ddot{w}_b}{\partial y} - \rho z\left[\frac{5}{3}\left(\frac{z}{h}\right)^2 - \frac{1}{4}\right]\frac{\partial \ddot{w}_s}{\partial y} \tag{3b}$$

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{z}}{\partial z} = \rho(\ddot{w}_b + \ddot{w}_s) \tag{3c}$$

The equilibrium equations in Eq. (3) can be rewritten in terms of stress resultants for a plate under a transversely distributed load q as shown in Fig. 1 by multiplying the first two equations by z and then integrating all three equations with respect to z, and applying several boundary conditions, i.e. the transverse shear stresses σ_{xz} and σ_{yz} equal to zero at $z = \pm h/2$ and the normal stress through the thickness $\sigma_z = 0$ at $z = -h/2$ and $\sigma_z = -q$ at $z = h/2$. The resulting equations are:

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = -I_2\frac{\partial \ddot{w}_b}{\partial x} \tag{4a}$$

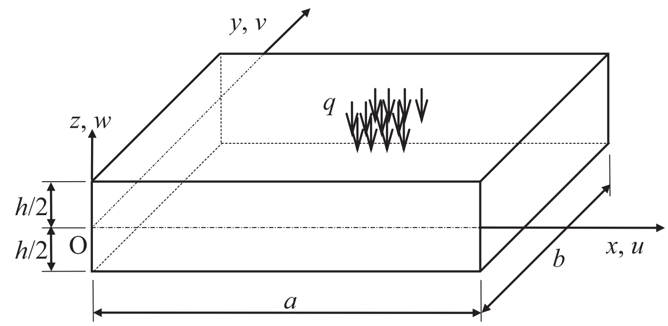

Fig. 1. Geometry and coordinates of a rectangular plate.

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{y}}{\partial y} - Q_y = -I_2\frac{\partial \ddot{w}_b}{\partial y} \tag{4b}$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = I_0(\ddot{w}_b + \ddot{w}_s) \tag{4c}$$

where the moments M, shear forces Q and mass inertias I are defined as

$$M_x = \int_{-h/2}^{h/2} z\sigma_x dz \tag{5a}$$

$$M_y = \int_{-h/2}^{h/2} z\sigma_y dz \tag{5b}$$

$$M_{xy} = \int_{-h/2}^{h/2} z\sigma_{xy} dz \tag{5c}$$

$$Q_x = \int_{-h/2}^{h/2} \sigma_{xz} dz \tag{6a}$$

$$Q_y = \int_{-h/2}^{h/2} \sigma_{yz} dz \tag{6b}$$

$$I_0 = \int_{-h/2}^{h/2} \rho dz \tag{7a}$$

$$I_2 = \int_{-h/2}^{h/2} \rho z^2 dz \tag{7b}$$

According to Shimpi [27], the moments and shear forces can be expressed in terms of the dependent unknowns (w_b, w_s) as

$$M_x = -D\left(\frac{\partial^2 w_b}{\partial x^2} + \nu\frac{\partial^2 w_b}{\partial y^2}\right) \tag{8a}$$

$$M_y = -D\left(\frac{\partial^2 w_b}{\partial y^2} + \nu\frac{\partial^2 w_b}{\partial x^2}\right) \tag{8b}$$

$$M_{xy} = -D(1-\nu)\frac{\partial^2 w_b}{\partial x \partial y} \tag{8c}$$

$$Q_x = A_s\frac{\partial w_s}{\partial x} \tag{9a}$$

$$Q_y = A_s\frac{\partial w_s}{\partial y} \tag{9b}$$

where

$$D = \frac{Eh^3}{12(1-\nu^2)} \text{ and } A_s = \frac{5Eh}{12(1+\nu)} \tag{10}$$