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Postbuckling of piezolaminated cylindrical shells with eccentrically/concentrically stiffeners surrounded by nonlinear elastic foundations

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ABSTRACT

A fully analytical approach on thermal and mechanical buckling and postbuckling of cylindrical shell surrounded on nonlinear elastic foundation is presented. The shell is composed of composite material, piezoelectric actuator, and eccentrically/concentrically isotropic stringers and rings. The equilibrium and compatibility equations of shell are derived based on Kirchhoff assumptions taking into account von Karman nonlinearity. Two types of simply-supported boundary conditions are considered as freely movable and immovable edges. The equations are solved by definitions of stress function and applying Galerkin method. Numerical examples are well verified with available data in the literature. Several parametric investigations are conducted to examine the effects of voltage, different stiffeners, lay-up configuration, and nonlinear elastic foundations on equilibrium paths.

1. Introduction

The studies of postbuckling of shells made by different materials attracted many structural investigators all over the world. Among the many materials that have significant attractions on the equilibrium paths of cylindrical shells, one of them is the piezoelectric layers. Piezoelectric sensors or actuators can be effectively used in traditional laminated composite structures due to the direct and converse effects [1–11]. On the other hand, cylindrical shells with stiffeners have good characteristics against different loads. In other word, the shells can be reinforced by stringers and rings to have high load carrying capacities [12–19]. Therefore there is a need to compare the effects of piezoelectric and stiffener on the buckling and postbuckling of laminated composite shells.

In the field of buckling and postbuckling of piezoelectric-composite cylindrical shells, Shen and Li [4] and Shen [6,7] performed different thermal and mechanical postbuckling of laminated composite cylindrical shells with embedded or surface mounted piezoelectric actuators based on classical shell theory. They applied a semi-analytical method to determine bifurcation points and postbuckling load-deflection curves, which is named as singular perturbation technique. Ganesan and Kadoli [5] presented active control of buckling loads of laminated cylindrical shells by piezoelectric actuators and sensors. The equations are formulated based on first-order shear deformation theory and a finite element method is conducted to solve coupled mechanical and electrical equations. Also, a finite element method is presented to determine buckling load of piezo-composite shells by Varelis and Saravanos [20,21]. In their works, mechanical formulations incorporate the mixed-field shear layerwise laminate theory. Recently, Sahmani et al. [22] and Mehralian et al. [23] studied critical loads of piezoelectric cylindrical shells under various loads based on classical and first-order theory, respectively. Besides above mentioned numerical and semi-analytical methods, there is only a reference to study fully analytical buckling of piezo-composite cylindrical shells. Mirzavand et al. [24] calculated closed form solutions of thermal buckling of laminated composite shells with two piezoelectric actuators based on classical shell theory. However, it still needs work to derive closed form solutions for buckling and especially postbuckling of piezo-composite shells.

Focusing on buckling and postbuckling of stiffened composite cylindrical shells in the literature reveals that experimental reports are widespread. Chenghu and Zhe [25] presented the experimental

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Nomenclature

x, y, z	cylindrical coordinates
L, R, h	length, mean radius, and thickness of the shell
d_{11s}, d_{22s}	width of stringer and ring
S_{11s}, S_{22s}	space between two stringers and rings
$h_s, 2h_c, 2h_a$	thickness of stiffener, composite, and piezoelectric
$k_0, k_1, k_2, K_0, K_1, K_2$	Winkler, Pasternak, and nonlinear parameters and dimensionless forms
$\varepsilon_{xx}, \varepsilon_{yy}, \gamma_{xy}$	strain components
$\varepsilon_{xx}^{(0)}, \varepsilon_{yy}^{(0)}, \gamma_{xy}^{(0)}$	strain components of mid-surface
$\varepsilon_{xx}^{(1)}, \varepsilon_{yy}^{(1)}, \gamma_{xy}^{(1)}$	curvatures
u_0, v_0, w_0	displacement components
k	number of ply in laminated composite
α	angle of fibers in a ply
$\sigma_{xx}, \sigma_{yy}, \tau_{xy}$	stress components
$\alpha_x, \alpha_y, \alpha_{xy}$	thermal expansion coefficients of a ply
$\Theta, \Delta T$	thermal load and uniform temperature rise
$\bar{Q}_{ij}(i,j=1,2,6)$	stiffness matrix of a ply
$Q_{ij}^{s}(i,j=1,2,6)$	stiffness matrix of stiffener
E_{stiff}	elastic modulus of stiffener
$Q_{ij}^{a}(i,j=1,2,6)$	Stiffness matrix of piezoelectric actuator
$\alpha_x^a, \alpha_y^a, \alpha_{xy}^a$	thermal expansion coefficients of piezoelectric actuator
e_{31}, e_{32}	electric stiffness of piezoelectric actuator
E_x, E_y, E_z, V_a	electric field components and voltage through the thickness direction
$N_x, N_y, N_{xy}, M_x, M_y, M_{xy}$	force and moment resultants
$N_x^T, N_y^T, N_{xy}^T, M_x^T, M_y^T, M_{xy}^T$	thermal force and moment resultants
$N_x^E, N_y^E, N_{xy}^E, M_x^E, M_y^E, M_{xy}^E$	electrical force and moment resultants
$A_{ij}, B_{ij}, D_{ij}(i,j=1,2,6)$	in-plane, coupled, and bending stiffness
$I_i^s, I_i^a(i=1,2,3)$	inertia moments of stiffener and piezoelectric
$\Phi, \xi_1, \xi_2, \xi_3$	stress function and its coefficients
$AQ_{ij}(i,j=1,2,6)$	inverse of total extensional stiffness
N_{x0}, F_x	pre-buckling force and compression
W	maximum deflection
$m, n, \bar{m}, \bar{n}$	number of half-waves and full waves in the axial and circumferential directions
$\beta_i(i=1,2,\ldots,6)$	functions in terms of $AQ_{ij}(i,j=1,2,6)$ and coupled stiffness
$AQ_x^T, AQ_y^T, AQ_{xy}^T$	thermal stiffness of both composite and piezoelectric layers

data for buckling load of composite cylindrical shell with internal stringers and rings. Yazdani and Rahimi [26] tested buckling load of cylindrical shell with two types of internal stiffeners i.e. lozenge and triangular. The experimental buckling of cylindrical shell with internal stringers and external rings under combined axial compression and torsion are presented by Bisagni and Cordisco [27,28]. Also, Ren et al. [29] presented an experimental set up with finite element modelling for buckling load of composite cylindrical shell with internal Kagome pattern grid. Among the other finite element methods, Shi et al. [30] and Rahimi et al. [31] presented buckling analysis of composite cylindrical shells with internal stiffeners.

Due to complexity and time consumption of semi-analytical and analytical methods, only a few researchers have attempted to extend formulations of stiffened composite shells. Wang et al. [32] and Kidane et al. [33] determined buckling loads of composite shells with internal isogrid stiffeners by Ritz method. Unfortunately, the constitutive equations of stiffeners are derived by the smeared method, which Talezadehlari and Rahimi [34] demonstrated that the smeared method is not applicable. Shen [12,14,15] and Zeng and Wu [17] presented buckling and postbuckling of composite shells with internal stringers and rings using perturbation technique. To the best of author's knowledge, there is not any analytical data about buckling and postbuckling of laminated cylindrical shell with eccentrically and concentrically stiffeners.

For elastic foundations, the buckling and postbuckling of laminated composite cylindrical shells surrounded by only two parameter elastic foundations can be found in many works, but there are very few studies on the behavior of beams, plates and shells on three parameter elastic foundations. Shen [35] presented postbuckling analysis of composite plates on softening nonlinear elastic foundations. Tornabene and Reddy [36] presented GDQ solution for static analysis of FGM and laminated shells on three parameter elastic foundations. Ghiasian et al. [37] studied dynamic buckling of FGM beams on three parameter elastic foundations. Zhang and Zhou [38] imported the nonlinear elastic foundation into postbuckling of FGM plates. Very most recently, Sofiyev et al. [39] and Sofiyev [40] studied vibration of composite cylindrical shells with three parameter and nonlinear Winkler elastic foundations, respectively.

The objective of this paper is to develop closed form solutions for thermal and mechanical buckling and postbuckling responses of piezoelectric-stiffener-composite shells for the first time in the literature. The laminated composite shell is covered by two piezoelectric actuators and two types of location of stiffener are considered i.e. eccentric and concentric. The total shell is surrounded by three parameter elastic foundation including Winkler, Pasternak, and nonlinear softening/hardening parameters. The equilibrium and compatibility equations are obtained on the basis of classical laminated shell theory. Then, the closed form expressions of postbuckling paths are obtained by applying the Galerkin procedure.

2. Fundamental equations

Consider a circular cylindrical shell as shown in Fig. 1. The length, mean radius, and total thickness of the shell are L, R, and h, respectively. The coordinate system of the shell is defined as (x, y, z) in which the origin of system is situated in the middle surface. The coordinate x and y are in axial and circumferential directions and z axis is normal and inward to the shell. In this article, we consider two types of configuration of structure. For model 1, two piezoelectric actuators are bonded to the outer and inner surfaces of laminated composite shell and then shell is reinforced by eccentrically stiffener (in external surface). For model 2, the piezoelectric-composite shell in previous model is reinforced by concentric stiffener. In such a way, the total thickness of these two models is same. In order to describe the geometric parameters of details, we have d_{11s} and d_{22s} which mean the width of stringers and rings, respectively, S_{11s} and/or S_{22s} are the distance between two parallel stringer and/or ring, h_s, $2h_c$, and $2h_a$ are the thickness of stiffener, composite, and piezoelectric layers.

According to classical shell theory, we can introduce the strains $(\varepsilon_{xx}, \varepsilon_{yy}, \gamma_{xy})$ in terms of strain components of middle surface $(\varepsilon_{xx}^{(0)}, \varepsilon_{yy}^{(0)}, \gamma_{xy}^{(0)})$ and curvatures $(\varepsilon_{xx}^{(1)}, \varepsilon_{yy}^{(1)}, \gamma_{xy}^{(1)})$ as follow [41]