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An analytical study of mechanical behavior of polypropylene/calcium carbonate composites under uniaxial tension and three-point bending

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A B S T R A C T

Analytic solutions of test curve are beneficial to analyze mechanical properties and to understand the reason behind mechanical behaviors of materials. Herein, two relationships about both average true strain and average true stress on necking were obtained. Further, the homogenizing constitutive equation was obtained according to the linear viscoelastic constitutive equation becoming the foundation of solving analytic solutions of material behavior. The applicability of analytic solutions in the entire test range was discussed and improved by experimental results of both uniaxial tension and three-point bending. Analytic solutions of test curve in this work conform well to the experiments. The moduli and strengths of polypropylene/calcium carbonate composites were predicted according to this model. This mathematical model indicates that mechanical behavior of linear viscoelastic material was the outcome of competition between stress-increasing of extension and stress-decreasing of relaxation. It is hoped that the empirical formulae of two routine test curves can bring convenience for an analytical study of mechanical behavior of linear viscoelastic materials.

1. Introduction

Polymeric materials are typical viscoelastic materials. In particular, the particle-reinforced composites are macroscopically isotropic and simple structure, which are linear viscoelastic materials characterized in terms of the integral representation of the Boltzmann superposition principle as well as the differential representation based on the generalized Maxwell or Kelvin model. Two kinds of constitutive equations are equivalent. The linear viscoelastic constitutive equation of tridimensional anisotropy is also developed to characterize different components based on the correspondence principle to the theory of elasticity [1]. Furthermore, the fractional viscoelastic constitutive equation is employed [2–5], which fewer parameters are required to represent material viscoelastic behavior than traditional integer-order models [4]. Mechanical behaviors of materials are understood by the macromechanics theory that has a set of complete equations [1,6]. Accordingly, the mechanical problems of different materials are discussed in detail by macromechanics theory, so does viscoelastic theory, for instance, dynamic problems [7–10], anisotropic problems [11–14] and non-linear problems [5,12,15–17]. Meanwhile, mathematical operations are usually difficult due to the complex boundary conditions or constructions in engineering, but some powerful numerical methods are developed to simulate the mechanical behaviors of materials under different situations, such as the finite difference method [18–21] or the finite element (FE) method [5,9,22–27]. Most engineering problems have been solved.

The most common ways of mechanical performances test of materials, the uniaxial tension and three-point bending, are a testing standard to obtain the material constants with stationary boundary conditions. If there are some analytical solutions of mechanical behaviors of material, it will be beneficial to execute test in detail. This is meaningful for analyzing what the role of parameters is understood based on various functional relations rather than numerical calculations.

In this paper, two relationships on necking were obtained from viscoelastic theory, which are the average true strain $\langle \varepsilon_x(t) \rangle$ and average true stress $\langle \sigma_x(t) \rangle$ related to the nominal strain ε_0 and nominal stress σ_0, respectively. Further, the homogenizing constitutive equation between $\langle \varepsilon_x(t) \rangle$ and $\langle \sigma_x(t) \rangle$ was obtained and became the foundation of solving geometrically nonlinear mechanical behavior question. The analytic solutions of test curve both uniaxial tension and three-point bending were solved by the differential equations. And then the applicability of analytic solutions

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was discussed emphatically via experimental results. The model was corrected by experiments and empirical formulae were built in two routine test to suit large deformed flexible polymeric materials. The moduli and strengths of polypropylene/calcium carbonate composites were predicted according to this model. The impact of different test rates to materials performance was discussed, which was convenient to convert among different test conditions. And the changing tendency of mechanical behavior in the entire test range influenced by parameters was shown. The establishment of the model will be beneficial to quantitative research the relationship among a series of ingredients of linear composites, such as guide formulation design of linear composites when the relationships among model parameters and the dosage of a series of raw material was known.

2. The homogenizing constitutive equation on necking

2.1. The definition of average true strain

Viscoelastic materials are mostly high ductile materials, which are produced large deformation when it suffers the sustained stress. The large deformation is characterized by geometric equation. Corresponding to the condition of uniaxial tension (Detailed process are shown in the Appendix A), the elongation Δl of a specimen is equal to

$$\Delta l = \int_0^{\Delta l} du = \int_0^{l_0} \left(\sqrt{2\varepsilon_x(x,y,z,t) + 1 - f^2(x,y,z,t)} - 1 \right) dx \qquad (1)$$

where $f^2 = (\partial v/\partial x)^2 + (\partial w/\partial x)^2$, ε_x is the true strain. There is $v = w = 0$ and $\varepsilon_x(x,y,z,t) = \varepsilon_x(t)$ before necking generated. Hence the Eq. (1) reduces as

$$\varepsilon_x(t) = \varepsilon_0(t) + \frac{1}{2}\varepsilon_0^2(t)$$

where $\varepsilon_0(t) = \Delta l/l_0$ is the nominal strain.

As is shown in part A of Fig. 1, however, $v, w \neq 0$ and ε_x is related to the coordinates in the place where necking occur. (Fig. 1 is the finite element numerical simulation) There is also a formula resemblance to the relation before necking generated, which was proven in Appendix A.

$$\langle \varepsilon_x(t) \rangle = \varepsilon_0(t) + \frac{1}{2}\varepsilon_0^2(t)$$

where $\langle \varepsilon_x(t) \rangle$ is called average true strain and $\langle \varepsilon_x(t) \rangle \in [\varepsilon_{min}, \varepsilon_{max}]$. ε_{max} is true strain of the smallest columnar shape in the specimen and ε_{min} is the biggest. Hence $\varepsilon_x(t) = \varepsilon_{min}(t) = \langle \varepsilon_x(t) \rangle = \varepsilon_{max}(t)$ when there is nonexistent necking.

2.2. The definition of average true strain

The volume variation ratio g is changing with the process of stretching as a function of the nominal strain

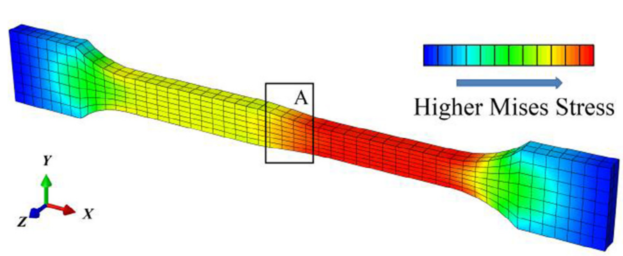

Fig. 1. The varying stress in the necking A otherwise well-distributed stress respectively (Draw in ABAQUS 6.14-2).

$$\frac{V_0}{V} = g(\varepsilon_0) \qquad (2)$$

Hence, the true stress is obtained based on Eq. (2) when the condition is uniaxial tension before necking generated

$$\sigma_x(t) = \frac{F}{A} = \frac{F(l_0 + \Delta l)g(\varepsilon_0)}{A_0 l_0} = \sigma_0(1 + \varepsilon_0)g(\varepsilon_0)$$

where A_0, l_0 are initial size of cross-sectional area and gauge length respectively. $\sigma_0 = F/A_0$ is the nominal stress.

After necking, there is also a formula resemblance to the relation before necking, which was proven in Appendix B.

$$\langle \sigma_x(t) \rangle = \sigma_0(1 + \varepsilon_0)g(\varepsilon_0)$$

where $\langle \sigma_x(t) \rangle$ is called average true stress and $\langle \sigma_x(t) \rangle \in [\sigma_{min}, \sigma_{max}]$. σ_{max} is the stress of the smallest columnar shape in the specimen and σ_{min} is the biggest. Hence $\sigma_x(t) = \sigma_{min}(t) = \langle \sigma_x(t) \rangle = \sigma_{max}(t)$ when there is nonexistent necking.

Some detailed investigations [28,29] that is volume variation process of materials during tensile are carried out. Assuming that the function $g(\varepsilon_0)$ is enough to form the Taylor's series, the higher order in $O(\varepsilon_0^2)$ can be omitted if the volume changed is not obvious in the finite tensile range. We say the linear variation, $g(\varepsilon_0) = 1 + \alpha\varepsilon_0$. Therefore

$$\langle \sigma_x \rangle = \sigma_0(1 + \varepsilon_0)(1 + \alpha\varepsilon_0)$$

2.3. The constitutive equation related to the average true strain and average true stress

The constitutive equation of linear viscoelastic materials does relate to $\varepsilon_x(t)$ and $\sigma_x(t)$ rather than $\langle \varepsilon_x(t) \rangle$ and $\langle \sigma_x(t) \rangle$. In this paper, the homogenizing constitutive equation on necking was proven according to the definition of the average true strain and average true stress based on the linear viscoelastic constitutive equation in Appendix C

$$\langle \sigma_x(t) \rangle + \sum_{i=1}^{m} p_i \frac{d^i}{dt^i} \langle \sigma_x(t) \rangle = \sum_{i=0}^{n} q_i \frac{d^i}{dt^i} \langle \varepsilon_x(t) \rangle$$

3. A uniaxial tension model and three-point bending model

3.1. The relationship between the nominal stress and nominal strain

It was expedient to consider the standard linear solid constitutive equation ($\langle \dot{\varepsilon}_x \rangle$ denotes $d\langle \varepsilon_x \rangle / dt$)

$$\langle \sigma_x \rangle + p_1 \langle \dot{\sigma}_x \rangle = q_0 \langle \varepsilon_x \rangle + q_1 \langle \dot{\varepsilon}_x \rangle \qquad (3)$$

that included first-order derivative and only one relaxation time but a satisfactory conclusion was still given out, which was illustrated by the experiments in this paper.

For uniaxial tension, $\varepsilon_0(t) = \dot{\varepsilon}_0 t$. $\dot{\varepsilon}_0$ is the strain rate of extension, $\dot{\varepsilon}_0 = v_0/l_0$. According to Section 2.1, it is easy to obtain

$$\langle \varepsilon_x \rangle = \dot{\varepsilon}_0 t + \frac{1}{2}\dot{\varepsilon}_0{}^2 t^2, \quad \langle \dot{\varepsilon}_x \rangle = \dot{\varepsilon}_0 + \dot{\varepsilon}_0{}^2 t \qquad (4)$$

Substituting Eq. (4) into the (3) and solving it (Details of solving process are shown in the Appendix D), we get

$$\langle \sigma_x \rangle = A_t \left[1 - exp\left(-\frac{\varepsilon_0}{B_t} \right) \right] + C_t \varepsilon_0^2 + D_t \varepsilon_0$$

$$\text{where } A_t = (1 - p_1\dot{\varepsilon}_0)(q_1 - p_1 q_0)\dot{\varepsilon}_0, \; B_t = p_1\dot{\varepsilon}_0, C_t = q_0/2, \; D_t = 2C_t + A_t/(1 - B_t). \qquad (5)$$