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A new doubly-curved shell element for the free vibrations of arbitrarily shaped laminated structures based on Weak Formulation IsoGeometric Analysis

Francesco Tornabene *, Nicholas Fantuzzi, Michele Bacciocchi

DICAM – Department, School of Engineering and Architecture, University of Bologna, Italy

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ABSTRACT

The main aim of the paper is to present a new numerical method to solve the weak formulation of the governing equations for the free vibrations of laminated composite shell structures with variable radii of curvature. For this purpose, the integral form of the stiffness matrix is computed numerically by means of the Generalized Integral Quadrature (GIQ) method. A two-dimensional structural model is introduced to analyze the mechanical behavior of doubly-curved shells. The displacement field is described according to the basic aspects of the general Higher-order Shear Deformation Theories (HSDTs), which allow to define several kinematic models as a function of the free parameter that stands for the order of expansion. Since an Equivalent Single Layer (ESL) approach is considered, the generalized displacements evaluated on the shell middle surface represent the unknown variables of the problem, which are approximated by using the Lagrange interpolating polynomials. The mechanical behavior of the structures is modeled through only one element that includes the double curvature in its formulation, which is transformed into a distorted domain by means of a mapping procedure based on the use of NURBS (Non-Uniform Rational B-Splines) curves, following the fundamentals of the well-known Isogeometric Analysis (IGA). For these reasons, the presented methodology is named Weak Formulation Isogeometric Analysis (WFIGA) in order to distinguish it from the corresponding approach based on the strong form of the governing equations (Strong Formulation Isogeometric Analysis or SFIGA), previously introduced by the authors. Several numerical applications are performed to test the current method. The results are validated for different boundary conditions and various lamination schemes through the comparison with the solutions available in the literature or obtained by a finite element commercial software.

1. Introduction

Most of physical phenomena can be modeled and simulated through a set of differential equations. The heath transmission, the motion of a fluid, as well as the mechanical behavior of a solid element, are all examples of physical problems ruled by differential equations [1–7]. These equations represent the mathematical models of the physical phenomena and are obtained by introducing the proper assumptions [1,4]. In general, such models are governed by complex equations defined in arbitrary domains. The main aim of the researchers who work in the wide field of computational mechanics is to find and develop a numerical technique able to solve these complex differential equations, whose solution cannot be achieved analytically. As highlighted in the works by Tornabene et al. [8,9], the various numerical approaches that can be used for this purpose can be classified according to the formulation that they aim to solve. The choice is performed between the strong and the weak formulations. When the strong formulation is considered, the governing differential equations are directly transformed into a system of discrete equations. The well-known Generalized Differential Quadrature (GDQ) method, developed by Shu [10,11] and broadly applied by the authors to deal with the structural analyses of plates and shells [12–65], represents an extremely efficient and reliable numerical tool able to provide accurate solutions to these kinds of problems based on the strong formulation. The authors used this numerical approach to deal with laminated composite plates and shells [12–24], functionally graded structures [25–35], and structural elements reinforced by

* Corresponding author.
E-mail address: francesco.tornabene@unibo.it (F. Tornabene).
URL: http://software.dicam.unibo.it/diqumaspab-project (F. Tornabene).

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nanoparticles [36–38]. Analogously, the same scheme coupled with higher-order shear deformation theories was employed to achieve the numerical solution to similar problems [39–51]. Finally, it should be noted that the GDQ method can be used to solve the strong formulation when the reference domain is characterized by arbitrary geometries [52–65]. In general, the strong formulation needs a higher order of derivation in comparison with the corresponding weak form, which is introduced to weaken (or reduce) the order of differentiability [3]. Indeed, a weighted-integral statement is introduced when the weak formulation is solved to provide an equivalent form to both the governing equation and the associated natural boundary conditions [66,67]. It should be highlighted that the weak formulation is taken into account when a Finite Element Method (FEM) is employed to achieve the numerical solution of a generic mathematical model. In general, the main idea on which a FEM is based is to approximate the unknowns of the problem by means of proper polynomials that interpolate the nodal values of the approximated functions. It should be noted that the approximating functions must be chosen so that the differentiability requirements needed by the weak formulation, as well as the boundary conditions, are satisfied. An exhaustive presentation of the weak form-based approaches can be found in the book by Reddy [4].

In this paper, the authors aim to present a new numerical approach based on the weak formulation of the governing equations to deal with the free vibration analysis of laminated doubly-curved shells [68–70]. In general, the topic of composite structures has always drawn the attention of many scientists due to the improved performances that the combination of two or more constituents can give to these structural elements in comparison with conventional mechanical configurations [71–80]. Analogously, the same topic has aroused the interest of many researchers who work in the computational mechanics field. In the current work, the integral form of the partial differential equations is computed through the Generalized Integral Quadrature (GIQ) method developed by Shu in the nineties [10,11]. The assumptions on which the mathematical model is based can be found in the general framework of Higher-order Shear Deformation Theories (HSDTs), since the displacement field is defined as a function of an arbitrary order of kinematic expansion [81–90]. For this purpose, it should be noted that the present structural model belongs to the widespread category of Equivalent Single Layer (ESL) approaches since both the mechanical and geometric parameters of the model are related to the shell middle surface [22,38,42,44,50]. As highlighted in the book by Kraus [91], the middle surface of a generic shell structure represents the reference domain for the governing equations. The unknown variables of the problem are given by the generalized displacements of the middle surface and they are approximated employing the Lagrange interpolating polynomials. Differently from the conventional weak form approaches such as the ones solved in FEM, the double curvature of the shell is embedded in the model. This means that each considered structure can be characterized by two radii of curvature that vary point by point.

A peculiar mapping procedure based on NURBS curves is introduced to define arbitrarily curved domains, following the basic aspects of the well-known Isogeometric Analysis (IGA) [22,38,59,64,92–97]. This approach was widely used to investigate the mechanical behavior of different structural elements, as highlighted in the papers [98–110]. For completeness purpose, it should be mentioned that the main features of these curves can be found in the works [111–128], as well as in the book by Piegl and Tiller [129]. It should be noted that the authors have presented the same kind of analysis in the work [22], where the strong formulation of the governing equations has been solved by the GDQ method. As a consequence, they named such method as Strong Formulation Isogeometric Analysis (or SFIGA). On the other hand, in this work the authors introduce for the first time a procedure based on the weak formulation. In order to distinguish these two approaches, the current methodology can be denoted as Weak Formulation Isogeometric Analysis (or WFIGA). The accuracy and validity of the present method is proven in this paper by the comparison with the results available in the literature or obtained by a finite element analysis performed through a commercial software. The solutions obtained in the work [22] are also shown for the sake of comparison.

2. Shell geometry

The difficulties related to the description of a generic doubly-curved surface characterized by variable radii of curvature can be overcome by means of the fundamentals of differential geometry [91]. According to this approach, the curved surface that represents the reference domain of the governing equations is described by the position vector $\mathbf{r}(\alpha_1, \alpha_2)$, which depends on the principal curvilinear coordinates α_1, α_2. Once this vector is introduced, it is possible to identify the coordinates of each point P' upon the middle surface of the shell in the global reference system $O\ x_1\ x_2\ x_3$. As it can be noted from Fig. 1, each point P within the three-dimensional shell element is defined by the following expression

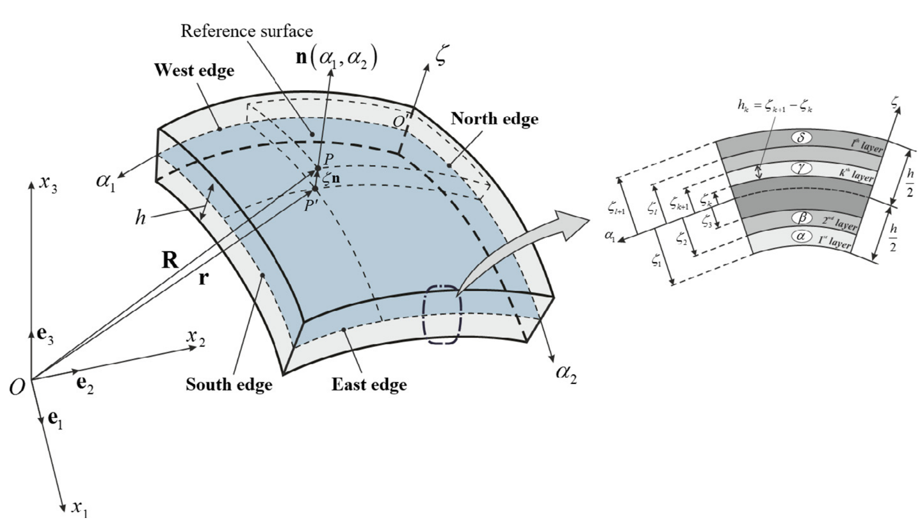

Fig. 1. Unmapped shell element and lamination scheme.