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Review

Bending, buckling and free vibration of laminated composite and sandwich beams: A critical review of literature

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ABSTRACT

Laminated composite and sandwich structures are lightweight structures that can be found in many diverse applications especially civil, mechanical and aerospace engineering. The rapid increase in the industrial use of these structures has necessitated the development of new theories that suitable for the bending, buckling and vibration analysis of composite structures. Many review articles are reported in the literature on laminated composite plates and shells in the last few decades. But, in the whole variety of literature very few review articles are available exclusively on laminated composite and sandwich beams. In this article, a critical review of literature on bending, buckling and free vibration analysis of shear deformable isotropic, laminated composite and sandwich beams based on equivalent single layer theories, layerwise theories, zig-zag theories and exact elasticity solution is presented. In addition to this, literature on finite element modeling of laminated and sandwich beams based on classical and refined theories is also reviewed. Finally, displacement fields of various equivalent single layer and layerwise theories are summarized in the present study for the reference of researchers in this area. This article cites 515 references and highlights, the possible scope for the future research on laminated composite and sandwich beams.

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1. Introduction

Beams, columns, and rods made up of a composite material are being widely used in the civil, mechanical, aeronautical and aerospace industries due to their attractive properties such as; high strength-and stiffness-to-weight ratio and their anisotropic material property. The beam members are mainly subjected to bending whereas columns and rods are mainly subjected to axial tension and compression. The shear deformation effects are more pronounced in the laminated composite and sandwich beams subjected to transverse loads. Therefore, bending, free vibration and buckling analysis of laminated composite and sandwich beams have received widespread attention in recent years. The various analytical and numerical methods based on beam theories have developed by the researchers for analysis of shear deformable laminated composite and sandwich beams. Many review articles are reported in the literature on laminated composite plates and shells in the last few decades [1]. But, in the whole variety of literature very few review articles are available exclusively on laminated composite and sandwich beams [2–7]. Kapania and Raciti [2,3] reviewed literature on bending, buckling, post-buckling, vibration and wave propagation analysis of laminated composite beams and plates using various displacement and stress based shear deformation theories. The mathematical formulation of some classical and higher order shear deformation theories is presented. Authors have also reviewed finite element models for the analysis of laminated beams and plates and provided suggestions for future research. Ghugal and Shimpi [4] presented a review of displacement and stress based refined theories for isotropic and anisotropic laminated composite beams. Merits and demerits of various equivalent single layer and layerwise theories are discussed. Marur [5] has thrown some light on distinct phases in the development of nonlinear vibration of beams. Gherlone [6] presented a brief review on zig-zag theories for the analysis of laminated composite and sandwich beams. It is concluded that the equivalent single layer theories are easy to implement and computationally affordable but, in order to correctly describe the mechanical behavior of laminated structures the displacement field needs to be enriched by a through-the-thickness piecewise linear contribution i.e. zig-zag function. Hajianmaleki and Qatu [7] presented a review of research done by various researchers on the vibration analysis of composite and smart beams. The review is based on classical and higher order shear deformation theories developed for the analysis of beams and numerical methods used for solving governing differential equations. The well-known books on laminated composite and sandwich beams have published by Reddy [8], Wang et al. [9] and Carrera, et al. [10].

Therefore, in the present study an attempt is made to review the research done by various researchers on isotropic, laminated composite and sandwich beams using different analytical and numerical methods based on classical and refined beam theories.

2. Bending of shear deformable beams

A great deal of research has been carried out since the last few decades to accurately assess the bending response of beams. Bending analysis of beams using two-dimensional elasticity theory is very complicated and this led to the development of refined shear deformation theories for beams which approximate the two dimensional solution into one dimensional solution with reasonable accuracy. Two classes of theories are available in the literature for the analysis of beams. In the first class of theories, the effects of shear deformation and rotary inertia are neglected whereas in the second class of theories, shear deformation and rotary inertia effects are considered.

2.1. Bending of isotropic beams

The most commonly used theory for the bending analysis of beams is classical beam theory (CBT) developed by Bernoulli-Euler [11–16] almost 300 years ago. Chronological development of this theory from the first attempt by Galileo in 1638 till 1856 by Barre de Saint Venant is given by Love [17], Timoshenko [18] and Todhunter and Pearson [19]. This theory neglects the shear deformation and rotary inertia effects; therefore, this theory is typically accurate for thin beams and is less accurate for thicker beams. Rayleigh [20] improved the classical theory by allowing the effect of rotary inertia of the cross-sections of the beam. Boley [21] studied an accuracy of the CBT for the beams of variable section. The stresses and deflections of the beam are found, on the basis of the two-dimensional theory of plane stress. The Deflection and stresses are obtained in the form of series using Airy-s stress function; the first term of each series is identical with the strength-of materials solution and the others represent the necessary correction to CBT. Timoshenko [22] developed a new beam theory which was considered as a refinement of the classical beam theory. This theory introduced first-order shear effects as well as effect of rotational inertia in the kinetic energy. Therefore, this theory is also known as the first order shear deformation theory or Timoshenko beam theory (TBT). The TBT violates the zero shear stress conditions on the top and bottom surfaces of the beam. A shear correction factor is required to appropriately represent the strain energy of shear deformation. TBT also suffers from the boundary condition paradox. Many articles are published on Timoshenko beam theory and shear correction factors [23–56]. Recently, Elishakoff et al. [57] presented historical development of TBT and the research available in the literature based on this hundred years old theory which considers the effects of transverse shear deformation and rotary inertia.

However, CBT and TBT do not take into account the non-classical influences such as cross-sectional warping, out-of-plane and in-plane deformations. The detail discussion on these influences is presented by Goodier [58]. Therefore, the higher order shear deformation theories are necessary to account for these non-classical influences. These higher order shear deformation theories are developed using the method of hypotheses, the method of expansion, the method of successive approximation and the mixed method. Donnell [59] and Boley and Tolins [60] have developed refined theories using the method of successive approximations for the bending of thick beams. Refined theories in which transverse shear stress is varying parabolically across the thickness of the beam and does not require problem dependent shear correction factor are called as parabolic shear deformation theories [61–80]. Irschik [81] has established an analogy between classical Bernoulli-Euler theory and refined beam theories of Levinson [68], Rehfield and Murthy [65] and by Rychtcr [74] using the principle of virtual work to increase the acceptance of higher order refined theories. The beams of rectangular cross-section with