Composite Structures 171 (2017) 562–575

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Three-dimensional modelling of heat conduction in laminated plates with the use of a two-dimensional numerical model

J. Jaśkowiec *, P. Pluciński

Cracow University of Technology, Faculty of Civil Engineering, Institute for Computational Civil Engineering, Warszawska 24, 31-155 Cracow, Poland

A R T I C L E I N F O

Article history:
Received 4 December 2016
Revised 9 February 2017
Accepted 10 March 2017
Available online 18 March 2017

Keywords:
Laminated composite plate
Finite element method
Transverse approximation

A B S T R A C T

This paper presents a two-dimensional numerical (2D) model for analysing the full three-dimensional (3D) problem of heat transport problems in laminated composite plates. A 2D finite element model is enough for good representation of a 3D problem since each single node in the mesh has enough degrees of freedom associated with particular layers of the laminated plate. The 2D numerical model is quite simple and effective and it allows considering multi-layered structures in which individual layers are made of different materials and the structure may consist of a combination of thick and very thin layers without numerical instabilities. The method presented in this paper is combined with a specially tailored postprocessing procedure so that results from the planar mesh can be visualised as 3D graphics. The paper is illustrated with a set of examples in which stationary and non-stationary problems are considered. In these examples modelling of the laminated glass with three glass panes and thin layers of bonding material is presented.

1. Introduction

The present work considers three-dimensional (3D) numerical modelling of heat conduction in laminated composite plates consisting of many laminate layers. It will suffice to achieve our objective if we only use a two-dimensional (2D) in-plane finite element mesh. The 2D numerical model presented in this paper is flexible and relatively easy to use. It can be applied to structures with an arbitrary number of layers and the layers may be thick or very thin. The number of layers and their properties (i.e. thickness, thermal material properties) are just the input to the 2D model. In spite of the fact that a 2D mesh is used in the analysis, full three-dimensional results are obtained. The proposed approach is applied for heat transport in laminated plates, but it may also find application in analysis of mechanical problems, such as plate bending, vibrations etc.

The in-plane approximation of temperature is based on a 2D finite element mesh, while in the transverse direction the approximation that is obtained is a 1D approximation based on Lagrange polynomials. In the weak formulation of the problem, the integration in the transverse direction is rewritten with the use of adequate Gauss integration rules. As a consequence, the considered problem is expressed using an integral equation which is based only on the planar integrals. This leads directly to a linear set of equations that are based on a 2D mesh but, on the other hand, the vector of degrees of freedom consists of temperatures at points located on a 3D structure. After the post-processing, which has been specially tailored for the proposed method, the results can be presented in full 3D fields of temperature or heat flux.

The laminated composite plate is one of the most popular topics of structural analysis research. Such plates are widely used in most civil or automotive engineering structures and their thermal or mechanical analysis is supported by a number of mathematical theories and numerous software implementations. The laminated plates modelling has remained in the scope of interest of many scholars, e.g. [1–13], and new research undoubtedly continues to be carried out. The interested reader may find en extensive survey of mechanics of laminated composite plates in [14]. Among many approaches to analyse laminated plates the layer-wise theory [15] is used by many authors, see e.g. [16–21]. Also the unified formulation for multilayered anisotropic composite plates and shells proposed in [22,23] is widely used in finite element analysis, e.g. [24–27].

However, laminated composite plates are relatively rarely analysed in 3D due to problems related to with discretizations. Spatial finite elements have to be adjusted to the laminates and it may lead to ill- conditioning of elements with high aspect ratio (relation of the in-plane dimension to the transverse dimension in the finite element). On the other hand, to reduce the high aspect ratio, the

* Corresponding author.
E-mail address: j.jaskowiec@L5.pk.edu.pl (J. Jaśkowiec).

http://dx.doi.org/10.1016/j.compstruct.2017.03.046

mesh needs to be quite dense, which in turn leads to computational inefficiency. Some scholars have considered using the finite element method (FEM) in the 3D plate analysis, [28]. However, 3D analysis is only expedient when three-dimensional effects are considered [29] or the layered plate structure needs to be modelled [30–32]. Sometimes, the mixed 2D/3D formulations are applied to plates to get more accurate results [33–36].

A similar idea of a 3D plate analysis on a 2D mesh has been proposed in [37,38] and also in [39,40] which discussed heat transport through laminate plates. In [37], the so-called scaled boundary finite element method (SBFEM) [41] was exploited to obtain the plate bending solution. The SBFEM provides a semi-analytical solution to the considered problem. The transverse solution for the plate is obtained analytically since there is a matrix function which is evaluated using Taylor expansion. In the approach presented in [40] the temperature along the layer of the laminate plate was approximated using linear or quadratic approximation. The integral along the layer thickness was calculated analytically. The Proper Generalized Decomposition was applied in [38,42,43] which allowed solving the fully 3D model by keeping the 2D characteristic computational complexity. In this approach, the plate-type spatial decomposition of displacement and temperature fields is used. In the decomposition the in-plane 2D solution is combined with the transverse 1D solution and in consequence, another 1D mesh is constructed, besides the planar 2D mesh. In the work in question, the method has been used to solve thermo-mechanical problems in a homogeneous plate, a laminate composite plate and honeycomb composites. The method has been subsequently developed in [44], where a piecewise fourth-order Lagrange polynomial along the thickness was applied. The present work is indebted to works [37–40], primarily to [38], yet it goes a step forward since the transversal solution has been completely excluded from the final numerical model. In paper [38], the combined 2Dx1D problem had to be solved, whereas in this paper a purely 2D problem is formulated for a 3D analysis.

The method proposed in this paper is also similar to component-wise (CW) model presented in the paper [45], which has been further applied for problems of free vibrations [46,47] or failure analysis in composite structures [48]. In CW model and in the proposed model the displacement function is decomposed and present as a multiplication of 1D and 2D functions, so it can be applied to many structures make of composites. In CW model the 1D decomposition component is chosen in the longitudinal direction. Whereas in this paper the 2D component is the in-plane surface and 1D goes into the transverse direction. Additionally, in this paper the closed formulas for planar finite elements is prepared, what means that only the integration over 2D mesh has to be performed. In CW model the 1D mesh is dominant. Furthermore, in this paper the method is complemented by postprocessing smoothing procedure and 3D visualisation technique. It results in excellent agreement with the standard 3D FEM analysis for heat conduction problems.

This paper is focused on the presentation of the numerical model for heat transport in a laminated composite plate and its numerical verification. It is assumed that the heat transfer is non-stationary, which entails the need to perform time integration and to sequentially solve a linear system of equations in order to get temperature increments. Each laminate layer in the transverse direction is approximated by the first, second or fourth order approximations. A global system of equations for the whole structure is subsequently developed from the assembling procedure. In the presented approach, very thick layers may be combined with very thin layers without numerical instabilities. The order of approximation in transverse direction can be adjusted to the layer thickness. For a thick layer, second or fourth order transverse approximation can be used while for a thin layer the first order will be sufficient. In such a situation, it is quite easy to model thin layers with bonding materials, e.g. polyvinyl butyral (PVB) or Ethylene Vinyl Acetate (EVA), that are used to construct laminated glass [49,50].

The approach proposed in this paper is first presented for an isotropic, homogeneous plate. It is subsequently extended to accommodate multi-layered structures with isotropic layers. The concept of in-plane and second order transverse approximations for a homogeneous plate is presented in Fig. 1. It is assumed that there are three in-plane finite element meshes on the lower S^l, middle S^m and upper S^u surfaces. Providing that temperatures on these meshes are known, using the second order Lagrange approximation along the plate thickness, we will obtain the temperature in the transverse direction as well. All the three in-plane meshes are in fact the same, so we may use only one mesh, on surface S^m, instead of three. The degrees of freedom on the single in-plane mesh consist of temperatures on the lower, middle and upper surfaces which means, in other words, that one node keeps

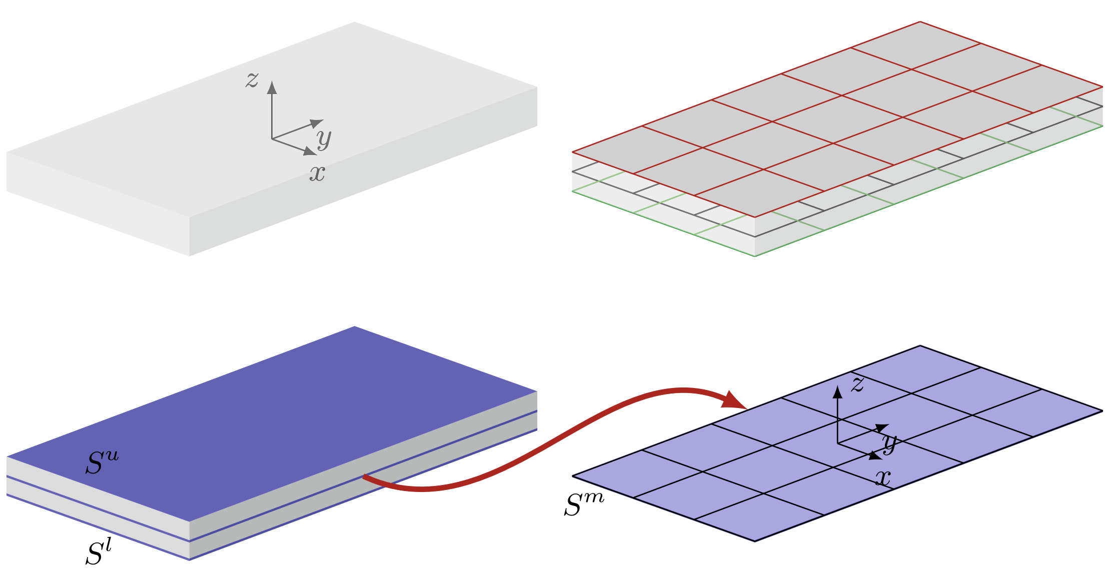

Fig. 1. Concept of in-plane and 2nd order transverse approximation for a homogeneous plate. Three surfaces are denoted: upper S^u, lower S^l and middle S^m.