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Nonlinear analysis of bending, thermal buckling and post-buckling for functionally graded tubes by using a refined beam theory

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ABSTRACT

Nonlinear bending, thermal buckling and post-buckling analysis for functionally graded materials (FGMs) tubes with two clamped ends by using a refined beam theory are investigated. The theory satisfies the traction-free boundary conditions on the inner and outer surfaces of the tube and also takes into account the transverse shear effects without artificially introducing shear correction factors. The material properties of FGM tubes are assumed to be temperature-dependent and vary in the radial direction. The asymptotic solutions of the FGM tubes under nonlinear bending and thermal post-buckling are solved by using a two-step perturbation method. The analytical solutions of Timoshenko beam and Euler beam are also presented. Detailed parametric studies are performed to investigate effects of inner-to-outer radius ratio, volume fraction as well as shear deformation on nonlinear bending, thermal buckling and post-buckling characteristics of the FGM tubes.

1. Introduction

Functionally graded materials (FGMs) are a new kind of inhomogeneous complex materials in which the microstructure, the composition and properties vary spatially through non-uniform distribution of reinforcement phase [1]. Due to the advantages of physical and chemical properties, FGMs has received great attention among scholars and has applied in many disciplines such as spaceflight, nuclear reactor, biomedicine, and nuclear industry [2,3]. As a new generation of composite materials, functionally graded materials has the unique capability to prevent delamination, alleviate thermal stress and eliminating stress concentration, which has more merits than other traditional complex material [2,3]. It has become a material of choice to be used in new structure. Many studies for FGM beams subjected to mechanical or thermal loading are available in the literature.

For FGM beams with rectangular cross sections, many scholars have conducted many studies on bending, buckling and vibration of FGM beams based on the model of Euler-Bernoulli beam, Timoshenko beam and higher order shear deformation beam [4–23]. As expected, the Euler-Bernoulli beam theory can only be suited for thin and long beams due to the lack of considering the effects of shear deformation. To account for the effect of transverse shear normal strain, Timoshenko beam was developed and was extensively applied to analyze the structural behavior, for Timoshenko beam model, Librescu and Stein [24] reported that the post-buckling load-deflection curves are sensitive to the selection of the shear deformation faction, so the shear correction factor needs to properly chosen. To mitigate for this limitation, Reddy [25] developed a simple higher order shear deformation theory, the advantage of this theory over Timoshenko beam theory is that the number of independent unknowns remain the same as in Timoshenko beam theory, but no correction factors are needed.

For linear and nonlinear analysis of beams with circular cross sections, the key issue is how to conduct a model in the governing equation. Huang and Li [26–28] developed a new model for beams with circular cross-section where shear deformation is taken into consideration, this beam model can satisfy traction-free boundary condition on the outer surfaces of columns. Based on that, Zhang and Fu extended [29] the beam model and proposed a refined higher-order shear deformation theory for tubes, this theory compared to Huang-Li beam model is that this model can degenerate to Huang-Li beam model when the inner radius of the tube is equal to zero. Based on this model, Zhong and Fu [30] discussed thermal

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post-buckling analysis of FGM tubes with two immovable simply supported ends based on higher-order beam model. This work was then extended to the cases of nonlinear bending and vibration of FGM tubes with two immovable simply supported ends by the same authors [31].

However, previous studies [29–31] only involved the nonlinear analysis of FGM tubes with simply supported ends, with only a few literature. In fact, the effect of boundary condition on analysis of structure is very significant, as indicated by Kadoli et al. [3]. Therefore, it is worth of investigating further on FGM tubes with different boundary conditions. The present paper intends to discuss nonlinear bending, thermal buckling and post-buckling of FGM tubes with two clamped ends. The material properties of FGM tubes are temperature-dependent and vary in the radical direction. The governing equation are based on Zhang-Fu beam model [29–31] and von Kármán type nonlinear strain-displacement relationship. The approximate solutions for nonlinear bending and thermal post-buckling are obtained by a two-step perturbation method [1,32]. The effects of inner-to-outer radius ratio, volume fraction index as well as shear deformation on nonlinear bending, thermal buckling and post-buckling characteristics of the FGM tubes are investigated.

2. Basic equations

A FGM tube of length L, inner radius R_i and outer radius R_0 with two clamped ends as shown in Fig. 1 is considered.

The tube is exposed to elevated temperature and subjected to transverse distributed load q. Let x, y and z be a set of coordinates with the x and y axes located at the corner of the FGM tube, and the z axes pointing downwards and perpendicular to x axis. The origin of the coordinate system is chosen at the corner of the tube on the mid-plane. Let u_1, u_2 and u_3 be the tube displacements parallel to a right-hand set of axes (x, y, z), respectively. Meanwhile, we also use polar cylindrical coordinates (x, r, θ) and the corresponding displacement vector (u, u_r, u_θ), where r is the distance from the mid-plane of the FGM tube, and θ is the angle down from y axis to r axis, which have following relations [26–28]:

$$y = r\cos\theta,\ z = r\sin\theta,\ u_r = v\cos\theta + w\sin\theta \tag{1}$$

Assume the FGM tubes are made from a mixture of ceramics and metals, the effective material properties P_f including Young's modulus E_f, shear modulus G_f, Poisson ratio v_f and thermal expansion coefficient α_f vary continuously in the radial direction, which can be expressed as

$$P_f(r,T) = (1-\Gamma)P_m(T) + \Gamma P_c(T),\ \text{with}\ \Gamma = [(r-R_i)/(R_0-R_i)]^N \tag{2}$$

where N is the volume fraction index which only take positive values $(0 \leqslant N \leqslant +\infty)$, P_m and P_c are corresponding temperature-dependent properties of metal and ceramic, respectively, and may be expressed as nonlinear function of temperature

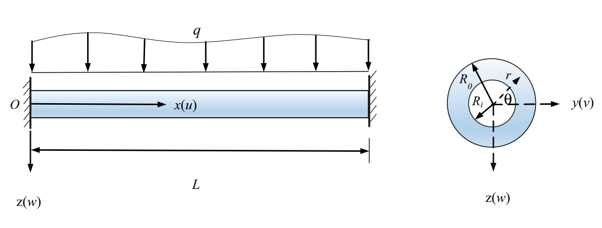

Fig. 1. Geometry and coordinates of A FGM tube with two clamped ends: (a) main view, (b) cross section view.

$$\begin{aligned} P_m &= P_0\left(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3\right), \\ P_c &= P_0\left(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3\right) \end{aligned} \tag{3}$$

in which, $T = \Delta T + T_0$. T (in Kelvin) is the temperature distribution through the tube, ΔT is the temperature increment from some reference temperature T_0 at which there are no thermal strains. As is customary, $T_0 = 300$ K. P_0, P_{-1}, P_1, P_2 and P_3 are the coefficients of Kelvin's temperature and unique to constituent materials. For simplicity, the temperature field T is assumed to be uniform.

Based on the beam model developed by Zhang and Fu [29–31], the displacement field can be expanded as the following form

$$\begin{aligned} u_1(x,y,z) &= u(x) + f(y,z)\tfrac{dw}{dx} + g(y,z)\varphi(x) \\ u_2(x,y,z) &= 0 \\ u_3(x,y,z) &= w(x) \end{aligned} \tag{4}$$

in which

$$\begin{aligned} f(y,z) &= z\left(R_0^2R_i^2r^{-2} - r^2/3\right)\left(R_0^2 + R_i^2\right)^{-1} \\ g(y,z) &= z + z\left(R_0^2R_i^2r^{-2} - r^2/3\right)\left(R_0^2 + R_i^2\right)^{-1} \end{aligned} \tag{5}$$

where $\varphi(x)$ is the mid-plane rotation of the normal about the y axis. It should be mentioned that, if $f(y, z) = 0$, Eq. (4) is for the case of Timoshenko beam's theory, which contain the same dependent unknowns (u, w, φ). If $f(y, z) = -z$, Eq. (4) is reduced to the case of Euler-Bernoulli beam' theory. If by setting the inner radius $R_i = 0$, Eq. (4) is for the case of the shear deformation theory for circular cylindrical beams developed by Huang-Li [26–28].

It is assumed that the present tubes deform within elastic regime and Hooke's law is valid. Note that w and φ are only a function of x, $f(y, z)$ is the function of y and z. Considering nonlinear von Kármán strain-displacement relationships, the normal strain ε_{xx} and shear stress strains γ_{xy}, γ_{xz}, γ_{xr} of the FGM tubes associated with the displacement field given in Eq. (4) are

$$\begin{aligned} \varepsilon_{xx} &= \tfrac{du_1}{dx} + \tfrac{1}{2}\left(\tfrac{du_3}{dx}\right)^2 = \varepsilon_x^{(0)} + f\varepsilon_x^{(1)} + g\varepsilon_x^{(2)}, \\ \gamma_{xy} &= \tfrac{\partial f}{\partial y}\gamma_{xz}^{(0)},\ \gamma_{xz} = \left(1 + \tfrac{\partial f}{\partial z}\right)\gamma_{xz}^{(0)},\ \gamma_{xr} = \gamma_{xz}\sin\theta + \gamma_{xy}\cos\theta \end{aligned} \tag{6}$$

in which

$$\varepsilon_x^{(0)} = \frac{du}{dx} + \frac{1}{2}\left(\frac{dw}{dx}\right)^2,\ \varepsilon_x^{(1)} = \frac{d^2w}{dx^2},\ \varepsilon_x^{(2)} = \frac{d\varphi}{dx},\ \gamma_{xz}^{(0)} = \left(\varphi + \frac{dw}{dx}\right) \tag{7}$$

When the thermal effect is taken into consideration, the constitutive relations of the tubes can be expressed as

$$\begin{aligned} \sigma_{xx} &= E_f(r,T)[\varepsilon_{xx} - \alpha(r,T)\Delta T],\ \tau_{xr} = G_f(r,T)\gamma_{xr} \\ \tau_{xy} &= G_f(r,T)\gamma_{xy} = \tfrac{E_f(r,T)}{2\left[1+v_f(r,T)\right]}\tfrac{\partial f}{\partial y}\gamma_{xz}^{(0)}, \\ \tau_{xz} &= G_f(r,T)\gamma_{xz} = \tfrac{E_f(r,T)}{2\left[1+v_f(r,T)\right]}\left(1 + \tfrac{\partial f}{\partial z}\right)\gamma_{xz}^{(0)} \end{aligned} \tag{8}$$

where $E_f(r, T)$ is Young's modulus, $G_f(r, T)$ the shear modulus, and $v_f(r, T)$ is Poisson ratio. The stress resultants and couples can be defined by

$$\begin{aligned} N_x &= \int_A \sigma_{xx}dA = A_0\varepsilon_x^{(0)} - N^T \\ M_x &= \int_A \sigma_{xx}fdA = A_1\varepsilon_x^{(1)} + A_2\varepsilon_x^{(2)} - M_x^T \\ P_x &= \int_A \sigma_{xx}gdA = A_2\varepsilon_x^{(1)} + A_3\varepsilon_x^{(2)} - P_x^T \\ Q &= \int_A \left[\tau_{xy}\tfrac{\partial f}{\partial y} + \tau_{xz}\left(1 + \tfrac{\partial f}{\partial z}\right)\right]dA = A_4\gamma_{xz}^{(0)} \end{aligned} \tag{9}$$