Composite Structures 169 (2017) 97–104

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

A substructure-based homogenization approach for systems with periodic microstructures of comparable sizes

Yang Chen [a], Leiting Dong [b], Bing Wang [c], Yuli Chen [b], Zhiping Qiu [b], Zaoyang Guo [b,a,*]

[a] Department of Engineering Mechanics, Chongqing University, Chongqing 400044, China
[b] School of Aeronautic Science and Engineering, Beihang University, Beijing 100191, China
[c] Center for Composite Materials, Harbin Institute of Technology, Harbin 150080, China

ARTICLE INFO

Article history:
Received 24 October 2016
Revised 31 December 2016
Accepted 17 January 2017
Available online 19 January 2017

Keywords:
Substructure approach
Periodic microstructure
Finite element
Homogenization

ABSTRACT

The classical homogenization method has been widely adopted to capture the effective behaviors of heterogeneous materials. However, when the characteristic length of the microstructure of the heterogeneous material is comparable to the size of the structure, the classical homogenization method is mathematically no longer valid. In this paper, a new substructure-based homogenization approach is proposed to predict the mechanical responses of systems with periodic microstructures of comparable sizes. A substructure element is developed to reconstruct the system with periodic microstructure of comparable size. It is verified that this substructure-based homogenization approach can accurately predict the mechanical responses of the system. Comparing with the full finite element analysis, the computational scale is dramatically decreased. After that, a simplified substructure element is developed by using less surface nodes in the "full" substructure element. The numerical results show that, with further significantly reduced computational cost, the third-order simplified substructure element can provide a good prediction of the responses of the system with periodic microstructure of comparable size.

1. Introduction

In recent years, the idea of homogenization has been widely adopted to estimate the effective behaviors of heterogeneous materials (e.g., composites, porous materials). One theoretical approach is to use theories of micromechanics to study the representative volume element (RVE) models and develop homogenized constitutive models to predict the effective responses of composites [1–6]. Another numerical homogenization approach is to use finite element method (FEM) to simulate the mechanical responses of the RVE models [7–10]. The RVE models are also widely used in analysis of system with periodic microstructures [11–13]. In the classical homogenization procedure, the obtained constitutive models are usually used to describe the stress-strain relation at the Gaussian integration points in finite element (FE) analysis of systems of heterogeneous materials. This implies that the classical homogenization approach is at the (macroscopic) material point scale. Therefore, mathematically the RVE size should be sufficiently smaller than the structure size but sufficiently larger than the characteristic length of the microstructure [14]. For example, Cricri and Luciano [15] suggested that the RVE models of cellular materials have at least 10 cells in each direction to get accurate effective parameters. Hence, the classical homogenization approaches based on RVE models cannot be adopted when the characteristic length of the microstructure of the heterogeneous material is comparable to the size of the structure (e.g., $L_{struture}/L_{material} < 10$). However, due to the novel manufacturing technologies such as 3D printing, the idea of integrated design of materials and structures leads to many structures with periodic microstructures of comparable sizes. In this paper, a substructure-based homogenization approach is developed to simulate the mechanical responses of systems with periodic microstructures of comparable sizes. In this substructure-based homogenization approach, the microstructures of the materials are considered as substructures of the structure because they are of comparable sizes. Therefore, in the substructure-based homogenization approach, the homogenization is at the structure scale rather than the material point scale. In the literature, Karpov et al. [16] adopted the substructures to analyze the static properties of finite repetitive structures, and some illustrative examples (i.e., truss bridge, clamped grid and honeycomb structures) are discussed with the substructure method. Li and Law [17] used substructures to reconstruct the system and study the response of a frame under the excitation force

* Corresponding author at: Institute of Solid Mechanics, Beihang University, 37 Xueyuan Road, Haidian District, Beijing 100191, China.
E-mail address: z-guo@foxmail.com (Z. Guo).

http://dx.doi.org/10.1016/j.compstruct.2017.01.050
0263-8223/

with wavelet domain method. Mencik [18] investigated the harmonic force response of one-dimensional periodic structures by analyzing the substructure with wave finite element method. Boldrin et al. [19] explored the dynamic behavior of gradient composite hexagonal honeycombs with a substructure-based component mode synthesis (CMS) method. Yu et al. [20] also adopted the CMS to study the mode shapes of the global structure with element-by-element model, which is used to update the large-scale structure. In these papers, the substructure concept is considered at the structure level, while in our approach, the substructure concept is also applied to the material level, i.e., the microstructure of the heterogeneous materials.

In this paper, the substructure element is developed to describe the microstructure of the heterogeneous materials. The global stiffness matrix of the RVE model is transformed to the elemental stiffness matrix of the substructure element by eliminating the interior degrees of freedoms (DOFs). Using the substructure elements, the simulation results obtained are identical to those from full FEM simulations, but the computational cost is much lower. Comparing to classical homogenization approaches, the substructure-based approach is particularly useful when the characteristic length of the heterogeneous material is comparable to the size of the structure. Moreover, to reduce the computational scale further, the simplified substructure element is proposed and it is shown that the third-order simplified substructure element can well predict the effective behavior of the systems with periodic microstructures of comparable sizes.

The structure of this paper is as follows: In Section 2, the substructure element is constructed, and the applications of the substructure-based homogenization approach are illustrated. Then the simplified substructure element is developed to further decrease the computational scale and it is applied to the analysis of the porous beams in Section 3. After that, some conclusion remarks are given in Section 4.

2. Substructure-based homogenization approach

In this section, the substructure element is developed to describe the microstructure of the heterogeneous material and its elemental stiffness matrix is derived using the concept of substructure. Then the substructure-based homogenization approach is applied to some examples to verify its accuracy and efficiency.

2.1. Stiffness of substructure element

Considering an RVE model of the microstructure of a heterogeneous material (an example shown in Fig. 1a), it is meshed and its mechanical response can be simulated using FEM by the following equation:

$$\mathbf{K}\mathbf{d} = \mathbf{P}, \tag{1}$$

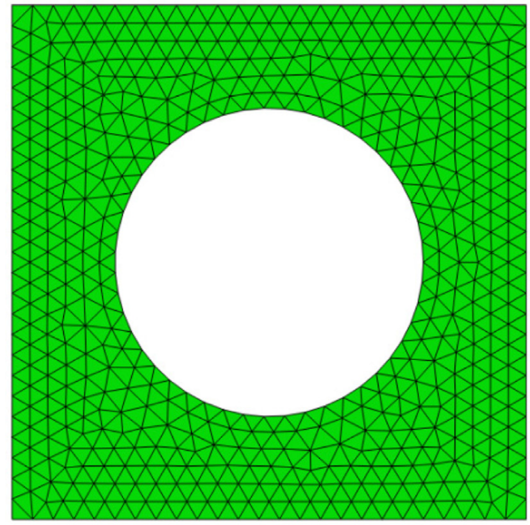
where $\mathbf{K}$ is the system stiffness matrix of the RVE model, $\mathbf{d}$ denotes the vector of nodal displacements, and $\mathbf{P}$ represents the vector of nodal forces. The nodes are further classified as the surface nodes and the interior nodes. Then Eq. (1) can be alternatively written as

$$\begin{bmatrix} \mathbf{K}_{ss} & \mathbf{K}_{si} \\ \mathbf{K}_{is} & \mathbf{K}_{ii} \end{bmatrix} \begin{bmatrix} \mathbf{d}_s \\ \mathbf{d}_i \end{bmatrix} = \begin{bmatrix} \mathbf{P}_s \\ \mathbf{P}_i \end{bmatrix}, \tag{2}$$

where subscript s and i denote the two kinds of nodes respectively. Hence, the response of the RVE can be approximated by the surface nodes only as

$$\mathbf{K}^*_{ss}\mathbf{d}_s = \mathbf{P}^*_s \tag{3}$$

where the elemental stiffness matrix of the substructure element is defined as


(a)

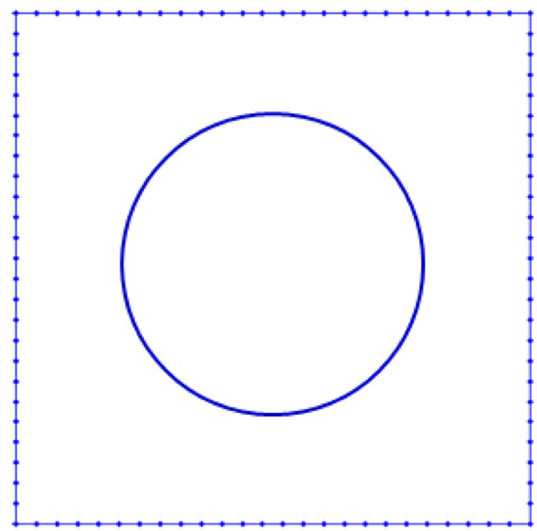

(b)

Fig. 1. The schematic diagrams of an RVE model (a) and the corresponding substructure element (b).

$$\mathbf{K}^*_{ss} = \mathbf{K}_{ss} - \mathbf{K}_{si}\mathbf{K}^{-1}_{ii}\mathbf{K}_{is}, \tag{4}$$

and the vector of equivalent surface nodal forces is computed as

$$\mathbf{P}^*_s = \mathbf{P}_s - \mathbf{K}_{si}\mathbf{K}^{-1}_{ii}\mathbf{P}_i. \tag{5}$$

Therefore, in comparison to the original RVE model, the dimension of the elemental stiffness matrix of the substructure element is reduced significantly due to the elimination of the DOFs of the interior nodes, which leads to much smaller computational scale. For instance, the RVE model of a unit square with a circular hole shown in Fig. 1(a) has 567 nodes and 987 triangular elements, which can be utilized in FEM to simulate its mechanical responses. The corresponding substructure element contains only 100 (surface) nodes (Fig. 1b), in which each edge is divided to 25 segments by 24 nodes on the edge. The number of elements also is reduced from 987 triangular elements to 1 single substructure element. It will be shown next that the substructure element can achieve exactly the same solution as the full FEM simulation of the detailed RVE model.