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Trigonometric-series solution for analysis of laminated composite beams

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ABSTRACT

A new analytical solution based on a higher-order beam theory for static, buckling and vibration of laminated composite beams is proposed in this paper. The governing equations of motion are derived from Lagrange's equations. An analytical solution based on trigonometric series, which satisfies various boundary conditions, is developed to solve the problem. Numerical results are obtained to compare with previous studies and to investigate the effects of length-to-depth ratio, fibre angles and material anisotropy on the deflections, stresses, natural frequencies and critical buckling loads of composite beams with various configurations.

1. Introduction

Composite laminated beams have been increasingly used in the various engineering fields for example in constructions, spacecraft, aircraft, mechanical engineering, etc. In order to predict accurately their structural responses, various beam theories with different approaches have been developed. These beam theories can be divided into three following categories: classical beam theory (CBT), first-order beam theory (FBT) and higher-order theory (HBT). A general review and assessment of these theories for composite beams can be found in [1–3]. It should be noted that CBT is only suitable for thin beams due to neglecting shear effect. FBT overcomes this adverse by taking into account this effect. However practically an appropriate shear correction is required. By using higher-order variation of axial displacement, HBT predicts more accurate than CBT and FBT, and importantly no shear correction factor is necessary. Therefore, this theory has been increasingly applied in predicting responses of composite beams.

For numerical methods, finite element method has been widely used to analyze composite beams [4–17]. For analytical approach, Navier solution is the simplest one, which is only applicable for simply supported boundary conditions [18–20]. In order to deal with arbitrary boundary conditions, many researchers developed different methods. Ritz-type method is commonly used [21–24]. Khdeir and Reddy [25,26] developed state-space approach to derive exact solutions for the natural frequencies and critical buckling loads of cross-ply composite beams. Chen et al. [27] also proposed an analytical solution based on state-space differential quadrature for vibration of composite beams. By using the dynamic stiffness matrix method, Jun et al. [28,29] calculated the natural frequencies of composite beams based on third-order beam theory. A literature review shows that although Ritz procedure is efficient to deal with static, buckling and vibration problems of composite beams with various boundary conditions, the research on this interesting topic is still limited.

The objectives of this paper is to develop a new trigonometric-series solution for analysis of composite beams with arbitrary lay-ups. It is based on a higher-order theory which accounts for a higher-order variation of the axial displacement. By using Lagrange equations, the governing equations of motion are derived. Ritz-type analytical solution with new trigonometric series is developed for beams under various boundary conditions. The convergence and verification studies are carried out to demonstrate the accuracy of the proposed solution. Numerical results are presented to investigate the effects of length-to-depth ratio, fibre angle and material anisotropy on the deflections, stresses, natural frequencies and critical buckling loads of composite beams.

2. Theoretical formulation

A laminated composite beam with rectangular section ($b \times h$) and length L as shown in Fig. 1 is considered. It is made of n plies

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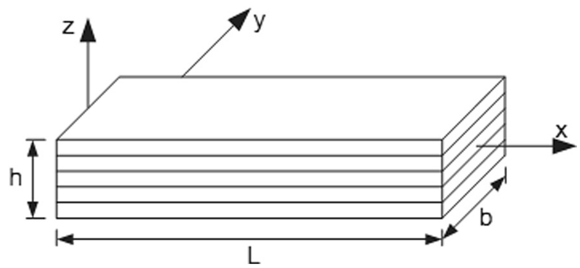

Fig. 1. Geometry of laminated composite beams.

Table 1
Trigonometric series for shape functions.

Boundary conditions	$\varphi_j\ (x)$	$\psi_j\ (x)$	$\xi_j\ (x)$
S–S	$\sin\frac{j\pi}{L}x$	$\cos\frac{j\pi}{L}x$	$\cos\frac{j\pi}{L}x$
C–F	$1-\cos\frac{(2j-1)\pi}{2L}x$	$\sin\frac{(2j-1)\pi}{2L}x$	$\sin\frac{(2j-1)\pi}{2L}x$
C–C	$\sin^2\frac{j\pi}{L}x$	$\sin\frac{2j\pi}{L}x$	$\sin\frac{2j\pi}{L}x$

Table 2
Three different boundary conditions of beams.

BC	$x = 0$	$x = L$
S–S	$w_0 = 0$	$w_0 = 0$
C–F	$u_0 = 0,\ w_0 = 0,\ \phi_0 = 0,\ w_{0,x} = 0$	
C–C	$u_0 = 0,\ w_0 = 0,\ \phi_0 = 0,\ w_{0,x} = 0$	$u_0 = 0,\ w_0 = 0,\ \phi_0 = 0,\ w_{0,x} = 0$

of orthotropic materials in different fibre angles with respect to the x-axis.

2.1. Kinetic, strain and stress relations

The displacement field of refined higher-order deformation theory ([30–32]) is given by:

$$u_1(x,z) = u_0(x) - zw_{0,x} + \left(\frac{5z}{4} - \frac{5z^3}{3h^2}\right)\phi_0(x)$$
$$= u_0(x) - zw_{0,x} + \Psi(z)\phi_0(x) \quad (1a)$$

$$u_3(x,z) = w_0(x) \quad (1b)$$

where u_0, ϕ_0 and w_0 are unknown mid-plane displacements of beam; Ψ is the shape function representing a higher-order variation of axial displacement; the comma indicates partial differentiation with respect to the coordinate subscript that follows.

The strain field of beams is given by:

$$\epsilon_{xx}(x,z) = u_{0,x} - zw_{0,xx} + \Psi(z)\phi_{0,x} = \epsilon_x^0 + z\kappa_x^b + \Psi(z)\kappa_x^s \quad (2a)$$

$$\gamma_{xz}(x,z) = \Psi_{,z}\phi_0 = g(z)\phi_0 \quad (2b)$$

where ϵ_x^0 and κ_x^b, κ_x^s are the axial strain and curvatures of the beam.

The stress of the k^{th}-layer is given by:

$$\sigma_{xx}^{(k)}(x,z) = \overline{Q}_{11}^{(k)}\left[\epsilon_x^0(x) + z\kappa_x^b(x) + \Psi(z)\kappa_x^s(x)\right] \quad (3a)$$

$$\sigma_{xz}^{(k)}(x,z) = \overline{Q}_{55}^{(k)}\gamma_{xz}(x,z) \quad (3b)$$

Table 3
Convergence studies for normalized mid-span displacements, fundamental frequencies and critical buckling loads of (0°/90°/0°) composite beams ($L/h = 5$, Material I, $E_1/E_2 = 40$).

BC	Number of series (m)							
	2	4	6	8	10	12	14	16
Deflection								
S–S	1.4978	1.4632	1.4685	1.4671	1.4676	1.4674	1.4675	1.4674
C–F	3.6160	4.0311	4.1035	4.1380	4.1499	4.1571	4.1604	4.1626
C–C	0.8696	0.9183	0.9274	0.9301	0.9311	0.9316	0.9319	0.9320
Fundamental frequency								
S–S	9.2084	9.2084	9.2084	9.2084	9.2084	9.2084	9.2084	9.2084
C–F	4.3499	4.2691	4.2473	4.2394	4.2359	4.2342	4.2332	4.2327
C–C	11.8716	11.6673	11.6269	11.6143	11.6093	11.6069	11.6056	11.6048
Critical buckling load								
S–S	8.6132	8.6132	8.6132	8.6132	8.6132	8.6132	8.6132	8.6132
C–F	4.7080	4.7080	4.7080	4.7080	4.7080	4.7080	4.7080	4.7080
C–C	11.6518	11.6518	11.6518	11.6518	11.6518	11.6518	11.6518	11.6518

Table 4
Normalized mid-span displacements of (0°/90°/0°) composite beam under a uniformly distributed load (Material II, $E_1/E_2 = 25$).

BC	Theory	L/h				
		5	10	20	30	50
S–S	Present	2.412	1.096	0.759	0.697	0.665
	Murthy et al. [11]	2.398	1.090	–	–	0.661
	Khdeir and Reddy [36]	2.412	1.096	–	–	0.665
	Vo and Thai (HBT) [14]	2.414	1.098	0.761	–	0.666
	Zenkour [37]	2.414	1.098	–	–	0.666
	Mantari and Canales [24]	–	1.097	–	–	–
C–F	Present	6.813	3.447	2.520	2.342	2.250
	Murthy et al. [11]	6.836	3.466	–	–	2.262
	Khdeir and Reddy [36]	6.824	3.455	–	–	2.251
	Vo and Thai (HBT) [14]	6.830	3.461	2.530	–	2.257
	Mantari and Canales [24]	–	3.459	–	–	–
C–C	Present	1.536	0.531	0.236	0.177	0.147
	Khdeir and Reddy [36]	1.537	0.532	–	–	0.147
	Mantari and Canales [24]	–	0.532	–	–	–