Composite Structures 160 (2017) 185–194

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

A strain gage technique for the determination of mixed mode stress intensity factors of orthotropic materials

Debaleena Chakraborty, D. Chakraborty *, K.S.R.K. Murthy

Mechanical Engineering Department, Indian Institute of Technology Guwahati, 781039, India

ARTICLE INFO

Article history:
Received 27 July 2016
Revised 28 September 2016
Accepted 16 October 2016
Available online 18 October 2016

Keywords:
Orthotropic
Mixed mode
Stress intensity factor
Strain gage
Radial location

ABSTRACT

A theoretical frame work is developed for strain gage based determination of mixed mode (K_I/K_{II}) stress intensity factors (SIFs) in slant edge cracked plate (SECP) made of orthotropic materials. Using three parameter strain series around the crack tip and appropriate stress functions, the present formulation shows that mixed mode SIFs in orthotropic materials could be determined using only four strain gages. A finite element based methodology is developed to determine the upper bound on the radial location (r_{max}) of strain gages ensuring accurate determination of SIFs. Proposed technique is applied to numerical simulation of $[0_2/90]_{2S}$ and $[0/\pm45/90]_S$ glass-epoxy SECP laminates to demonstrate accurate determination of mixed mode SIFs by placing the gages within r_{max}. Results from the present work provide clear guidelines in terms of number of strain gages and their suggested locations for accurate determination of mixed mode SIFs in orthotropic materials.

1. Introduction

Fracture mechanics based analysis of orthotropic materials has been an important area of research in general and the pioneering work towards the development of fracture mechanics studies of orthotropic materials was started by Irwin [1] in 1962. Various other researchers have taken these works further ahead [2–7]. Due to the limitation of analytical methods in dealing with general crack propagations and inconsistent geometrical and boundary conditions, several numerical and experimental techniques were developed by various researchers for solving fracture mechanics problems of orthotropic materials under mode I and mixed mode loading. Various numerical methods such as modified crack closure technique (MCCT) and displacement correlation technique (DCT) by Kim and Paulino [8], extended finite element method (XFEM) by Asadpoure and Mohammadi [9] and strong formulation finite element method (SFEM) based on generalized differential quadrature (GDQ) by Fantuzzi et al. [10] etc. to name a few have been employed for accurate prediction of SIFs of orthotropic laminates. In experimental analysis of anisotropic materials various techniques have been reported for measurement of fracture parameters. Baik et al. [11] used the method of caustics to obtain SIFs in orthotropic materials under mode I and mixed mode static loading conditions. Yao et al. [12] studied the stress singularities of mode I crack tip in orthotropic composites using the reflective caustic technology. Mojtahed and Zachary [13] devised a method for obtaining mode I SIF in orthotropic materials by using photoelastic technique. Ju and Liu [14] have used the technique of coherent grading sensing (CGS) interferometry to experimentally determine SIF for cracked composite panels (unidirectional graphite-epoxy) under bending (plane-stress deformation). Khanna et al. [15] used the technique of transmission photoelasticity to determine the SIF for cracks in composites. Mogadpalli and Parameswaran [16] investigated on the application of digital image correlation (DIC) to determine SIF for cracks in orthotropic composites.

The use of strain gages also forms an important segment in assessment of fracture parameters because of their economic and handling feasibility. However, various factors such as high strain gradients, finite gage size, 3D effects posed limitations in accurate determination of SIFs using strain gages. The single strain gage based determination of mode I SIF (K_I) of isotropic materials proposed by Dally and Sanford [17] in 1987 (DS technique) took care of most of the aforementioned difficulties by employing three parameter strain series equations. Consequently, various other researchers reported strain gage based methods for the determination of SIFs for either isotropic or orthotropic materials [18–25]. Amongst them, Shukla et al. [21] were the first to propose a technique for the determination of K_I of orthotropic materials employing two parameter strain field expressions around the crack tip. Khanna and Shukla [22] formulated the use of strain gages in

* Corresponding author.
E-mail addresses: debaleena@iitg.ernet.in (D. Chakraborty), chakra@iit.ernet.in (D. Chakraborty), ksrkm@iitg.ernet.in (K.S.R.K. Murthy).

http://dx.doi.org/10.1016/j.compstruct.2016.10.044
0263-8223/

accurate estimation of dynamic SIFs provided the gages are pasted within the singularity dominated zone (SDZ). Chakraborty et al. [24,25] were the first to propose an extension of the DS technique to orthotropic laminates with single-ended and double-ended cracks.

Frequent occurrence of mixed mode conditions can be attributed to the orientations which a crack generally makes with the loading direction. Researchers have numerically verified the partition of fracture modes on rigid interfaces in orthotropic laminated double cantilever beams (DCB) under general loading conditions that include crack tip bending moments, axial and shear forces [26,27]. In spite of frequent occurrence of mixed mode conditions, very few works have been reported with regard to strain gage based determination of mixed mode SIFs (K_I, K_{II}) even in case of isotropic materials. For example, Dally and Berger (DB technique) [28,29] were the first to propose an approach involving a strain series with more number of parameters than that proposed in the DS technique thus enabling the strain gages to be located at distances further away from the crack tip. Dorogoy and Rittel [30] employed a three strain gage rosette to measure K_I and K_{II} by considering a three parameter strain series. Sarangi et al. [31] proposed modified DB technique incorporating more number of parameters in the strain series for determination of K_I and K_{II} of isotropic materials.

Strain gage based applications require strain gages to be placed at locations away from the crack tip and the measured strains are then equated with suitable analytical expressions to extract the SIFs. However, the extent of radial distance of the strain gage from the crack tip is important as it dictates whether the selected analytical expression can be represented by the measured strains. Moreover, 3D effects and high strain gradients affect the accuracy of the measured strains if the strain gage(s) are placed very near to the crack tip [17,21,32]. In case of orthotropic materials, the radial distance of the strain gages should be at least equal to the thickness of the plate to avoid the 3D effects [21]. Sarangi et al. [26] estimated the permissible extent of the radial locations of strain gages ensuring accurate determination of mixed mode SIFs in isotropic materials. Chakraborty et al. [24,25] reported estimation of valid radial locations and orientation for a strain gage ensuring accurate estimation of K_I in orthotropic laminates. Small gage length gages are used to minimize the strain gradients effect [17,29].

Therefore, while development of suitable methodology for strain gage based determination of SIFs is important, it is also extremely important that the locations and orientations of the strain gages are known apriori ensuring accurate determination of SIF. Literature review reveals that though there are number of papers on determination of SIFs using strain gages and minimizing the number of strain gages for isotropic materials, there are only a counted few for orthotropic materials. In addition, analysis of notched/cracked composites has been an important area of research from the view point of delamination initiation from such notches/cracks under loading. Even though there has been numerical work on prediction of delamination initiation from such notches/cracks [33–35], in spite of the fact that strain gage based determination of SIFs is simple [36], there have been few works available in this direction [21–23]. One of the reasons for this is the fact that unlike isotropic materials, there has been lack of proper theoretical development for orthotropic materials supporting strain gage based determination of SIFs and providing guidelines for exact number of strain gages required and their locations facilitating accurate measurement of SIFs. Only recently, based on Irwin's stress function Chakraborty et al. [24] extended the DS technique [17] and proposed an appropriate theoretical development for measurement of K_I using a single strain gage. But till date, there has been no theoretical development reported for providing guidelines in measurement of mixed mode SIFs (K_I/K_{II}) for orthotropic materials using minimum number of strain gages and their recommended location, whereas the same has been developed for isotropic materials [29,31]. Therefore, the present paper aims at developing a suitable analytical framework for determination of mixed mode (I/II) SIFs using minimum number of strain gages in an orthotropic laminate. It also aims to propose a methodology to determine the maximum radial location ($r_{\max}$) within which the strain gages should be placed to ensure accurate determination of K_I and K_{II} using a three parameter strain series. Numerical simulations performed to substantiate the theoretical development are also presented which will provide guidelines and recommendations in terms of number of strain gages to be used and their optimal radial locations ensuring accurate values of K_I and K_{II} for conducting strain gage based experiments for a given configuration. The paper is organized as follows; the theoretical formulations are presented in Section 2. Section 3 gives detailed analysis of the numerical simulations and the corresponding results. Section 4 presents the concluding remarks.

2. Theoretical formulation

2.1. Strain gage techniques for determination of K_I and K_{II}

In 2D bodies, mixed mode loading indicates simultaneous occurrence of opening mode (mode I) and shearing mode (mode II) for which both K_I and K_{II} are required to describe the conditions near the crack tip. Employing the Westergaard's approach, Irwin [1,2] suggested stress functions for symmetric and skew-symmetric loading for orthotropic materials. For orthotropic materials, the present work uses the modified mode I stress function proposed by Shukla and co-workers [21] in line with the generalized Westergaard approach [37] as

$$F_I = \frac{1}{2}\left\{\mathrm{Re}\overline{\overline{Z_I}}(z_1) + \mathrm{Re}\overline{\overline{Z_I}}(z_2)\right\} - \frac{\beta}{2\alpha}\left\{\mathrm{Re}\overline{\overline{Z_I}}(z_1) - \mathrm{Re}\overline{\overline{Z_I}}(z_2)\right\} - \frac{\beta}{2\alpha}\left\{\mathrm{Re}\overline{\overline{Y_I}}(z_1) - \mathrm{Re}\overline{\overline{Y_I}}(z_2)\right\} \tag{1}$$

For mode II, the modified stress function as per generalized Westergaard approach [37] may be written as

$$F_I = \frac{1}{2}\left\{\mathrm{Re}\overline{\overline{Y_{II}}}(z_1) + \mathrm{Re}\overline{\overline{Y_{II}}}(z_2)\right\} - \frac{1}{2\alpha}\left\{\mathrm{Im}\overline{\overline{Z_{II}}}(z_1) - \mathrm{Im}\overline{\overline{Z_{II}}}(z_2)\right\} - \frac{1}{2\alpha}\left\{\mathrm{Re}\overline{\overline{Y_{II}}}(z_1) - \mathrm{Re}\overline{\overline{Y_{II}}}(z_2)\right\} \tag{2}$$

where $\overline{Z}_{I,II}(z_i)$ and $\overline{\overline{Z}}_{I,II}(z_i)$ are the first and second integrals with respect to $z_i(i = 1,2)$ of a complex function $Z(z_i)$ and $z_i(i = 1,2)$ is given by

$$z_1 = x + iy_1 = x + i(\beta + \alpha)y = r_1 e^{i\theta_1} \quad \text{and}$$
$$z_2 = x + iy_2 = x + i(\beta - \alpha)y = r_2 e^{i\theta_2} \tag{3}$$

and, $Y_{I,II}(z_i)$ is another analytical function used in conjunction with the standard Westergaard stress function $Z(z_i)$. Additionally, other parameters are defined as

$$\begin{aligned}
&\tan\theta_1 = (\beta+\alpha)\tan\theta; \quad \tan\theta_2 = (\beta-\alpha)\tan\theta \\
&r_1^2 = r^2(\cos^2\theta + (\beta+\alpha)^2\sin^2\theta); \quad r_2^2 = r^2(\cos^2\theta + (\beta-\alpha)^2\sin^2\theta) \\
&2\beta^2 = \frac{a_{66}+2a_{12}}{2a_{11}} + \sqrt{\frac{a_{22}}{a_{11}}}; \quad 2\alpha^2 = \frac{a_{66}+2a_{12}}{2a_{11}} - \sqrt{\frac{a_{22}}{a_{11}}}; \\
&a_{11} = \frac{1}{E_L}; \quad a_{12} = \frac{-\nu_{LT}}{E_L}; \quad a_{22} = \frac{1}{E_T}; \quad a_{66} = \frac{1}{G_{LT}};
\end{aligned} \tag{4}$$