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3D modeling of arbitrary cracking in solids using augmented finite element method

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ABSTRACT

A three dimensional (3D) augmented finite element method (AFEM) for modeling arbitrary cracking without the need of additional degree of freedom (DoFs) or phantom nodes is presented. Four or three internal nodes are employed to explain displacement jump due to the weak and strong discontinuity. In this method, damage and discontinuity are treated from a weak discontinuity to a strong one without additional degree of freedom and without explicit representation of the crack. A fully condensed elemental equilibrium equations as mathematical exactness in the piece-wise linear sense is explicitly derived within AFEM formulation. The method is implemented in ABAQUS 4-node tetrahedron user element with a local crack tracking method for crack path detection. Through some numerical examples, it is shown that the 3D AFEM can accurately and efficiently crack initiation and propagation.

1. Introduction

Accurate assessment of the structural integrity involves the development of complex progressive damage analysis with high-fidelity. Since standard finite element method is not suitable to model strong discontinuity and crack, advanced finite elements and numerical methods are developed to explicitly take into account the cracking and damage in the material, e.g., the generalized finite element method (GFEM), extended finite element method (XFEM) [1–8], phantom node method (PNM) [9–14], augmented finite element method [15–18], and meshless methods [19–25].

The generalized finite element, extended finite element and phantom node methods have been developed based on theory of the partition of unity and in essence introduce additional degree of freedom to account for arbitrary cracking. In the case of individual crack, these methods are mesh independent and effective. The shortcomings in these methods are a) the computational cost is very expensive due to additional degree of freedom, b) multiple crack interactions have to be established in the framework of these methods despite some recent articles to deal with multiple crack interactions [5,26,27]. In this regard approaches such as phase field [28–30] and embedded discontinuity [31–36] are developed to cope with arbitrary interacting cracks. Other emerging methods for failure analyses include the regularized FEM (RxFEM) of Iarve et al. [37] and the continuum–decohesion FEM by Waas and co-workers [38].

Phase field model introduces an additional nodal DoF to approximate a fracture surface with a continuous phase-field parameter and does not need to algorithmically trace fracture. However, the method is mesh-dependent and requires an extremely refined mesh to resolve the sharp discontinuity across the crack surfaces [39]. In embedded discontinuity method, special shape functions are used to account for the discontinuous crack displacements within an element [35]. Though, the special care must be taken for constructing shape functions orthogonality property to avoid spurious deformation or serious stress locking.

Meshless method which is based on interaction of each node with all its neighbor is another alternative to cope with arbitrary cracking problems in solids. The discrete nature of these mesh-free methods makes it easier in handling multiple crack interaction problems. Some recent advances in meshless method are the use of extrinsically enriched methods based on the partition of unity (PoU) theory [20–22] and the use of weight function enrichment [24,25].

A crack tracing method is a major part of modeling arbitrary cracking and is of particular challenge in simulation of 3D solid's failure. Many algorithms for tracing crack path have been developed [39–54]. Four common crack tracking methods are available including the local tracking method, the non-local tracking method, the global tracking method, and the level set methods. Advantages and shortcomings of the tracking methods are

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summarized in Table 1. More details on crack tracking algorithms are referred to Jager et al. [54]

It is noted that most of the above mentioned tracking methods (non-local, global, level set) may not be suitable for heterogeneous materials such as laminated or textile composites. Hence, we apply local tracking algorithm to trace crack path.

In the current paper, we seek to extend 2D augmented finite element method (AFEM), which has been proven to be able to account for multiple, arbitrary cracks and crack interactions in solids with much improved numerical efficiency [15–17] to account for 3D crack evolution in solids. The AFEM lies in the category of the embedded discontinuity method without employing discontinuous shape functions.

The remainder of this paper is organized as follows: After a short review of the problem statement and governing equations in Section 2, we briefly discuss the finite element discretization within the frame-work AFEM scheme in Section 3. Section 4 discusses the local crack tracing algorithm used in this study. Then, several numerical examples will be presented as compared with other works in Section 5. Finally, Section 6 concludes the paper with major highlights and numerical achievements.

Table 1
Summary of crack tracking methods.

Tracking method	Remarks
Local method	The crack surface is largely determined by the local elements immediately ahead of a crack front, subjecting to possible constraints from its neighboring crack points and surfaces [7,42,43]. The crack surface is of C^0 continuity, the computational cost is relatively low *Drawback*: The method may have difficulty in modeling non-planar crack
Non-local method	The crack surface is based on a least-squares fit to extend the existing crack surface as smoothly as possible [45,46]. Crack surface has less spurious zick-zack-type crack surfaces *Drawback*: The computational cost of this method is high, the crack surface may deviate from the real path, the complexity of implementation is relatively high.
Global method	An additional equation (heat conduction like) is introduced to track the crack front and provide *iso*-surface for crack [47,48]. The method is computationally robust and the crack surface the outcome of the solution of additional equation *Drawback*: It is computationally expensive due to the extra DoFs from heat conduction equation and requires a judicial choice of an initial boundary condition which is not always obvious for multiple cracking problems or different material interfaces.
Level set method	Signed distance functions is used to describe the crack surfaces [51–53] *Drawback*: There are some issues with freezing the crack surfaces as a crack grows, and inadequacy of finite element mesh for accurately solving the differential equations

2. Problem statement

Assume the 3D domain Ω of Fig. 1 is cut by a discontinuity into two sub-domains of Ω^+ and Ω^-. The discontinuity is assumed to be a cohesive crack with interface of $\Gamma_c = \Gamma_c^+ \cup \Gamma_c^-$. For $\Gamma_c^+ = \Gamma_c^-$, the discontinuity is weak and the interface is connected, while for strong discontinuity two surfaces are separated. The $\mathbf{f}$ and $\bar{\mathbf{u}}$ are prescribed traction and displacement on boundary of Γ_f, and Γ_u, respectively. The strong form of equilibrium equations along with boundary conditions are

$$\begin{array}{llll} \mathrm{Div}(\boldsymbol{\sigma}^+) = 0 & (\forall \mathbf{x} \in \Omega) & \mathrm{Div}(\boldsymbol{\sigma}^-) = 0 & (\forall \mathbf{x} \in \Omega^-) \\ \boldsymbol{\sigma}^+ \cdot \mathbf{n}^+ = \mathbf{f}^+ & (\forall \mathbf{x} \in \Gamma_F^+) & \boldsymbol{\sigma}^- \cdot \mathbf{n}^- = \mathbf{f}^- & (\forall \mathbf{x} \in \Gamma_F^-) \end{array} \quad (1)$$

where $\mathbf{n}^+$ and $\mathbf{n}^-$ are the outward normal of discontinuity surfaces, and $\boldsymbol{\sigma}^+$ and $\boldsymbol{\sigma}^-$ are the stresses in subdomains.

From the stress continuity across the discontinuity boundary, it follows.

$$\begin{array}{llll} \mathbf{u}^+ = \bar{\mathbf{u}}^+ & (\forall \mathbf{x} \in \Gamma_u) & \mathbf{u}^- = \bar{\mathbf{u}}^- & (\forall \mathbf{x} \in \Gamma_u^-) \\ \mathbf{t}^+ = \boldsymbol{\sigma}^+ \cdot \mathbf{n}^+ = -\mathbf{t} & (\forall \mathbf{x} \in \Gamma_c^+) & \mathbf{t}^- = \boldsymbol{\sigma}^- \cdot \mathbf{n}^- = \mathbf{t} & (\forall \mathbf{x} \in \Gamma_c^-) \end{array} \quad (2)$$

where $\mathbf{t}^+$ and $\mathbf{t}^-$ are the tractions along the discontinuity surfaces and $\mathbf{u}^+$ and $\mathbf{u}^-$ are the displacement fields in Ω^+ and Ω^-, respectively.

The traction is a function of the relative displacements ($\mathbf{t} = \mathbf{t}(\Delta\mathbf{u})$) between Γ_c^+ and Γ_c^-, where the relative displacement is $\Delta\mathbf{u} = \mathbf{u}^+ - \mathbf{u}^-$.where $\mathbf{u}^+$ and $\mathbf{u}^-$ are the displacement fields in Ω^+ and Ω^-, respectively. The constitutive law for traction-separation is a piece-wise linear in this study (See Appendix A).

The constitutive law and kinematic equations for subdomain Ω with the assumption of small strain and elastic behavior are written as

$$\begin{aligned} &\boldsymbol{\sigma}^+ = \mathbf{C}^+ : \boldsymbol{\varepsilon}^+ \ (\text{in } \Omega^+) \qquad \boldsymbol{\sigma}^- = \mathbf{C}^- : \boldsymbol{\varepsilon}^- \ (\text{in } \Omega^-) \\ &\boldsymbol{\varepsilon}^+ = \boldsymbol{\varepsilon}^+(\mathbf{u}^+) = \left[\nabla\mathbf{u}^+ + (\nabla\mathbf{u}^+)^{\mathrm{T}}\right]/2 \ (\text{in } \Omega^+) \\ &\boldsymbol{\varepsilon}^- = \boldsymbol{\varepsilon}^-(\mathbf{u}^-) = \left[\nabla\mathbf{u}^- + (\nabla\mathbf{u}^-)^{\mathrm{T}}\right]/2 \ (\text{in } \Omega^-) \end{aligned} \quad (3)$$

where $\mathbf{C}^+$ and $\mathbf{C}^-$ are the material stiffness tensors of the two subdomains traversed by the discontinuity, respectively. They are identical for homogeneous materials and different for heterogeneous materials.

The strong form of Eq. (3) can be written into a weak form using the principle of virtual work.

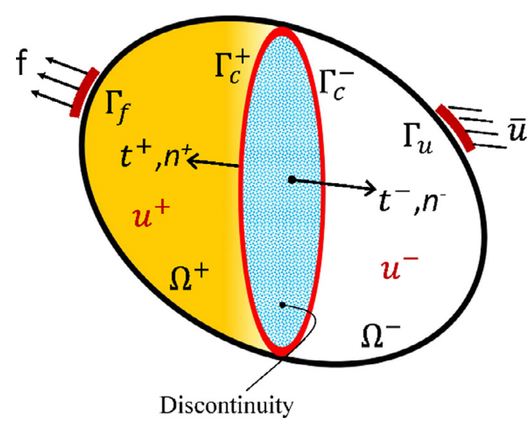

Fig. 1. Elastic body configuration with an arbitrary discontinuity.

$$\begin{aligned} &\int_{\Omega^+} \boldsymbol{\sigma}^+ : \boldsymbol{\varepsilon}^+(\mathbf{u}^+) d\Omega + \int_{\Gamma_c^+} \mathbf{t}(\Delta\mathbf{u}) \cdot \delta(\mathbf{u}^+) d\Gamma = \int_{\Gamma_F^+} \mathbf{f}^+ \cdot \mathbf{u}^+ d\Gamma \qquad \forall \mathbf{u}^+ \in u \\ &\int_{\Omega^-} \boldsymbol{\sigma}^- : \boldsymbol{\varepsilon}^-(\mathbf{u}^-) d\Omega - \int_{\Gamma_c} \mathbf{t}(\Delta\mathbf{u}) \cdot \delta(\mathbf{u}^-) d\Gamma = \int_{\Gamma_F^-} \mathbf{f}^- \cdot \mathbf{u}^- d\Gamma \qquad \forall \mathbf{u}^+ \in u \end{aligned} \quad (4)$$

3. AFEM formulation for A 3D 4-node tetrahedron

In this section we describe how to augment a 3D tetrahedron element with only regular nodes and DoFs. More details can be found in the work of Yang and co-workers [15–18]. A 4-node tetrahedron element is chosen to demonstrate the AFEM scheme (Fig. 2). As cut by a cohesive crack, there are two possibilities for tetrahedron cut including a) a tetrahedron sub-domain and a wedge sub-domain (Fig. 2b), b) two wedge sub-domains (Fig. 2c). Regular or external nodes and internal nodes are shown in cut element of Fig. 2. The crack front always resides at element boundaries during its propagation [17,18] and it is also assumed that