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Optimizing the natural frequencies of axially functionally graded beams and arches

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ABSTRACT

In this paper a new design methodology is presented to optimize the natural frequencies of axially functionally graded beams and arches by tailoring appropriately their material distribution. Functionally Graded Materials (FGMs) are deemed to have an advantageous behavior over laminated composites due to the continuous variation of their material properties yet in all three dimensions which alleviate delamination, de-bonding and matrix cracking initiation issues. The design of FGM structures is ideally fitted to optimization techniques, as the optimum material composition is derived by varying the relative volume fraction of the constituent materials. In this study the Differential Evolution (DE) is employed for optimizing the natural frequencies of axially FG beams and arches. The evaluation of the objective function requires the solution of a free vibration problem of an arch with variable mass and stiffness properties. The arches are modeled using a generic curved beam model that includes both axial (tangential) and transverse (normal) deformation and the problem is solved using the analog equation method (AEM) for hyperbolic differential equations with variable coefficients. The Mori-Tanaka homogenization scheme is adopted in this investigation for the estimation of the effective material properties. Several beams and arches are analyzed, which illustrate the design method and demonstrate its efficiency. In this work, and without restricting the generality, the FGM is comprised of steel and aluminum. The volume fraction of steel is assumed to follow two alternative distributions, namely a four-parameter power law distribution (FGM-1) or a five-parameter trigonometric distribution (FGM-2). In all cases three model problems are examined. We seek the material distribution that the FGM structure vibrates with (i) the maximum fundamental frequency, (ii) the minimum mass and the fundamental frequency greater than a prescribed value and (iii) the minimum mass and frequencies which lie outside certain frequency bands.

1. Introduction

In this work a new design methodology is presented to optimize the natural frequencies of axially functionally graded beams and arches by tailoring appropriately their material distribution. The philosophy behind the concept of Functionally Graded Material (FGM) is to construct a heterogeneous composite which performs as a single-phase material, by unifying the best properties of its constituent phase materials. FGMs are deemed to have an advantageous behavior over laminated composites due to the continuous variation of their material properties yet in all three dimensions which alleviate delamination, de-bonding and matrix cracking initiation issues. The design of FGM structures is ideally fitted to optimization techniques, as the optimum material composition is derived by varying the relative volume fraction of the constituent materials.

The endeavor to control the natural frequencies of a structure or structural element is a challenging and demanding task. By varying the mass and stiffness properties the design engineer seeks the structure to vibrate in accord to certain predefined criteria. This can be achieved via different types of structural optimization such as topology, shape or size [1], the detailed presentation of which is not the scope of this research. Besides the previous optimization techniques, the newly emerged FG materials have driven several researchers to address the so-called material volume fraction optimization. Qian and Batra [2] used a genetic algorithm together with a meshless local Petrov–Galerkin method and a higher-order deformable plate theory to find the compositional profile of

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a two-constituent cantilever plate so that either the first or the second natural frequency is maximum. In their work the volume fraction was assumed to vary according a simple power law along the thickness and longitudinal direction. Goupee and Vel [3] also used a genetic algorithm to optimize the natural frequencies of functionally graded beams employing the element free Galerkin Method to analyze the free vibration problem. They treated the beam as a plane stress problem using a piecewise bicubic interpolation of volume fraction specified at a finite number of grid points. In 2014, Alshabatat and Naghshineh [4] using an optimizer based on a genetic algorithm, determined the optimal profiles of volume fraction of beam constituent materials with the goal of maximizing the fundamental frequency which was computed via the Finite Element Method (FEM). In that paper they also presented two novel laws for describing the volume fractions of a FGM beam. The first one was a complex power law and the second one a trigonometric law.

A more complex optimization procedure for controlling the natural frequencies of FG materials has been proposed by some scholars. In this case, the simultaneous structural topology and material optimization has been addressed as a better alternative to the simple shape or material composition design. Rubio et al. [5], in 2011, presented a technique in order to achieve maximum and/or minimum vibration amplitudes at certain points of a structure, finding simultaneously the topology and material gradation function, whilst in 2014, Taheri and Hassani [1] developed a fully isogeometrical approach for the simultaneous shape and material composition design of FG structures to optimize their eigenfrequencies.

Although there have been a significant number of research papers on the analysis of the free vibration of FG arches [6–14], to the authors' knowledge, no work has been done on the design of axially FG arches for optimal natural frequencies. In our work the Differential Evolution (DE), a powerful metaheuristic algorithm, is employed for optimizing the natural frequencies of axially FG beams and arches. Nowadays, metaheuristic algorithms have emerged as one of the best ways for solving complex optimization problems. These algorithms are usually inspired by evolution, swarm intelligence or physical phenomena principles and their widespread use is justified by a number of important advantages such as easy implementation, lack of dependency on gradient or other problem-specific information and good performance with global search characteristics [15].

The evaluation of the objective function requires the solution of the free vibration problem of an arch with variable mass and stiffness properties. The arch is modeled using a generic curved beam model that includes both axial (tangential) and transverse (normal) deformation, and is also able to account for variable mass and stiffness properties, as well as elastic support or restraint. The resulting dynamic governing equations of the circular arch are formulated in terms of the displacements and solved using an efficient integral equation method [18–20]. Moreover, for the estimation of the effective material properties of two-phase FG materials several homogenization methods have been proposed. The two foremost are the self-consistent [21] scheme and the Mori-Tanaka [22] scheme. The former has been originally introduced to compute the mechanical response of polycrystals and takes into account the interaction of the matrix and the grains using Eshelby's solution, while the latter has been proposed by Tanaka and Mori for composite materials involving only two phases and, it is assumed that the inclusions in the RVE, experience the matrix strain as the far-field strain in the Eshelby theory [23]. In this investigation the Mori-Tanaka [22] homogenization scheme is adopted for the estimation of the effective material properties. The method works well in instances where the microstructure consists of clearly defined matrix and particulate phases [3].

Several beams and arches are analyzed, which illustrate the design method and demonstrate its efficiency. In this work, and without restricting the generality, the FGM is comprised of steel and aluminum. The volume fraction of steel is assumed to follow two alternative distributions, namely a four-parameter power law distribution (FGM-1) or a five-parameter trigonometric distribution (FGM-2). In all cases three model problems are examined. We seek the material distribution that the axially FGM structure vibrates with (i) the maximum fundamental frequency, (ii) the minimum mass and the fundamental frequency greater than a prescribed value and (iii) the minimum mass and frequencies which lie outside certain frequency bands.

2. Modelling and numerical formulation

2.1. Governing equations

Consider a plane curved beam the cross-sections of which are orthogonal to a plane curve (centroid axis) that belongs to the xz plane (see Fig. 1a). A curvilinear abscissa s spans the curved beam's centroid axis which undergoes the combined action of the distributed loads $p_t = p_t(s)$ and $p_n = p_n(s)$ acting in the tangential and normal direction, respectively (see Fig. 1b). The curved beam may have a cross-section with variable properties, that is, the axial $EA(s)$ and bending stiffness $EI(s)$ vary due to heterogeneous linearly elastic material $E = E(s)$.

The differential equations of equilibrium are derived by considering the equilibrium of an elementary section in projections onto the tangential t and the normal n directions and in moments with respect to one of the beam's ends [24]. Taking also into account the inertia and damping forces, we arrive at the following equations of motion [20]

$$-m\ddot{u} - c\dot{u} + \left[EA\left(u_{,s} + \frac{w}{R}\right)\right]_{,s} = -p_t \tag{1}$$

$$-m\ddot{w} - c\dot{w} - \left[EI\left(w_{,ss} + \frac{w}{R^2}\right)\right]_{,ss} - \frac{1}{R}\left[EA\left(u_{,s} + \frac{w}{R}\right) + \frac{EI}{R}\left(w_{,ss} + \frac{w}{R^2}\right)\right] = -p_n \tag{2}$$

where $m = m(s) = \rho A(s)$ is the mass density per unit length, c is the coefficient of viscous damping, R is the radius of the arch; $u(s)$ and $w = w(s)$ are the tangential and normal displacements, respectively. In addition, the subscript s preceded by comma denotes partial differentiation with respect to the curvilinear abscissa s.

The boundary conditions of the problem, that can include elastic support or restraint, are of the form

$$a_1 u(0,t) + a_2 N(0,t) = 0 \tag{3}$$

$$\bar{a}_1 u(l,t) + \bar{a}_2 N(l,t) = 0 \tag{4}$$

$$\beta_1 w(0,t) + \beta_2 Q(0,t) = 0 \tag{5}$$

$$\bar{\beta}_1 w(l,t) + \bar{\beta}_2 Q(l,t) = 0 \tag{6}$$

$$\gamma_1 \theta(0,t) + \gamma_2 M(0,t) = 0 \tag{7}$$

$$\bar{\gamma}_1 \theta(l,t) + \bar{\gamma}_2 M(l,t) = 0 \tag{8}$$

where $a_k, \bar{a}_k, \beta_k, \bar{\beta}_k, \gamma_k, \bar{\gamma}_k \, (k = 1, 2)$ are given constants and $\theta = -w_{,s} + u/R$ is the slope of the cross-section. It is apparent that all types of the conventional boundary conditions (clamped, simply supported etc.) can be derived from Eqs. (3)–(8) by specifying appropriately these constants.

The stress resultants appearing in the aforementioned equations are specified as [20]