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An equivalent continuum multiscale formulation for 2D geometrical nonlinear analysis of lattice truss structure

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ABSTRACT

In this work, an equivalent continuum multiscale formulation is presented for the geometrical nonlinear analysis of the structures with lattice truss materials. This formulation is established by combining the extended multiscale finite element method and the co-rotational approach. Firstly, the lattice truss unit cell is equivalent to a continuum coarse element by using a numerical constructed interpolation function in the local coordinate system. Then the tangent stiffness matrix of this coarse element is derived by employing the basic idea of the co-rotational approach in the global coordinate system. Thus, the global nonlinear equilibrium equations of the structure at the macroscopic level can be solved by using the general displacement control algorithm to capture the equilibrium path with multiple critical points. After performing all of the incremental steps and the iterative steps on the macroscopic scale, the microscopic information, such as the displacement, stress and strain, can be obtained easily by virtue of the aforeconstructed numerical interpolation functions once again. In addition, several numerical examples are carried out to study the effects of the layout and size of unit cell, investigate the sensitivity of coarse-scale meshes and verify the validation and efficiency of the presented multiscale formulation.

1. Introduction

Over the past few decades, the lightweight materials and structures are more and more applied in the aerospace industries and civil engineering areas with the development of manufacturing technologies [1,2]. As a kind of lightweight materials, lattice truss materials (as shown in Fig. 1) have received considerable attention due to their inherent advantages [3–5], such as high stiffness/strength-weight ratio [6]. As a result of the rapid development of three-dimensional printing and material preparation technologies, the construction of lattice truss materials and structures become much easier in recent years [7–11]. A lattice can be constructed by a large periodic truss bar with pin-jointed nodes. The lattice structures can be assembled by many periodic unit cells which are composed of some elementary truss bars. This type of structure has lower mass and higher rigidity than traditional ones. Such structures have been widely applied in civil engineering and they also have great potential to construct the ultra-large space structures.

The finite element method (FEM) is usually employed to analyze the problems of such lattice structures. However, these lattice structures usually have multiscale features, i.e., the minimum characteristic size of the lattice truss material is much smaller than the macroscopic size of the lattice structure. This is to say that each lattice structure contains numerous elementary truss bars. Therefore, lots of computational resources will be consumed when using the traditional FEM to solve the problems of lattice structures. What's worse is that the FEM will be failure to do this for some ultra-large lattice structures due to the limitations of the computer capability and CPU time. To solve this problem, many continuum models were developed since the lattices look like continuum media when they become large [13–15]. Moreau and Caillerie [15] developed a continuum model based on the homogenization method for 2D large displacement analysis of lattice truss structure. Tollenaere and Caillerie [16] studied a two-dimensional quasi-periodic lattice truss structure and presented a continuous model derived from the periodic continuous medium homogenization. Burgardt and Cartraud [17] provided a general approach to determine the equivalent beam properties of beam-like lattice trusses based on the energy equivalence. Elsayed and Pasini [18] investigated the structural design of the microscopic lattice

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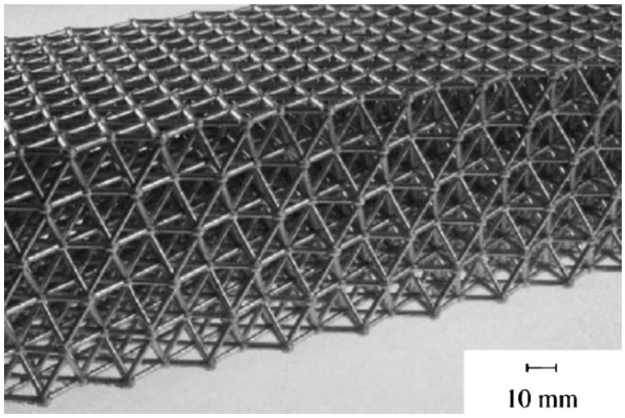

Fig. 1. Photograph of the lattice truss material [12].

architecture with octet-truss cell and the multiscale design of an axially loaded member manufactured of this type of cellular solid.

In this paper, an alternative equivalent continuum model, based on the extended multiscale finite element method (EMsFEM) [19–21] and the co-rotational formulation of geometrical nonlinear problems [22–24], is developed to analyze the geometrical nonlinear problem of lattice truss structure. The EMsFEM was firstly employed to solve the problems of heterogeneous materials in computational solid field by Zhang and his co-workers. This multiscale method has been successfully applied in simulating the linear and nonlinear problems of heterogeneous solid materials [25–27], the mechanical behaviors of bionic nastic materials [28,29], the thermo-mechanical coupling problems of multiphase solid materials [30,31] and the consolidation behaviors of saturated porous media [32,33]. The basic idea of this method is to construct multiscale base functions on the sub-grid domain. These functions can capture and bring the microscopic material information to the macroscopic scale for reducing the overall computational cost efficiently and significantly. In addition, a downscaling computation technique is also proposed for recovering the microscopic results of sub grids from the macroscopic scale solutions of overall coarse-scale mesh. On the other hand, the co-rotational formulation [34–36], as one of common existing formulations to solve the geometrically nonlinear problems [37], is developed for the equivalent coarse-scale continuum element in this work. Within this co-rotational formulation, there are two sets of coordinate systems: local and global ones. The local framework is fixed on the coarse-scale continuum element and moves with the rigid body translation and rotation of this element, while as the global one is fixed and unchanged all the time during the whole deformation processes of structure. This co-rotational formulation divides the whole movement of a finite element into two parts, where one is the rigid body translation and rotation while the other one is the deformation in the local coordinate system. After obtaining the multiscale base functions by the EMsFEM in the local coordinate system, the periodic lattice truss unit cells can be equivalent to a continuum element, whose effective local stiffness matrix and external load vector will be calculated easily and naturally by using the above-mentioned multiscale base functions. Then the tangent stiffness matrix and internal force vector of each equivalent continuum element can be obtained in the global coordinate system via the co-rotational formulation. Thus the global tangent stiffness and external load vector of the equivalent coarse-scale mesh of whole structure can also be assembled naturally. After that, the equilibrium iterative processes for each load incremental step will be performed to find the complex load-displacement equilibrium path by employing the frequently used iteration control algorithms [38], such as the arc-length control methods (ALCM), the generalized displacement control method (GDCM), the work control method (WCM) and the orthogonal residual procedure (ORP). Due to the excellent merit at capturing complex nonlinear behavior at both load and displacement limit points, the GDCM is employed in present work. Finally, the desired microscopic results of any truss element within the unit cell can be recovered easily via the downscaling calculation. To verify the validation and effectiveness of this proposed method, some typical numerical examples are investigated by using both this equivalent multiscale continuum method on the coarse-scale mesh and the traditional FEM on the full-scale mesh.

2. Briefly review of the EMsFEM

In EMsFEM, the bridge between microscopic and macroscopic levels is built through a multiscale shape function, which is usually obtained by solving a boundary value problem on a unit cell domain numerically. Then, the microscopic material properties could be captured and brought to the macroscopic scale for reducing the computational cost significantly. After the macroscopic calculation, the constructed multiscale shape function could be used once again to get the microscopic deformation information of each unit cell. As shown in Fig. 2, both the upscaling and downscaling computation processes can be achieved by virtue of the above-mentioned multiscale shape function.

3. Macroscopic equivalent quantities of the lattice truss unit cell

Generally speaking, three kinds of boundary condition (BC) can be employed for constructing the numerical multiscale shape function, i.e., linear, periodic and oversampling oscillating BCs, which are usually effective for homogeneous, periodic and completely heterogeneous materials, respectively. In this work, the periodic BC is adopted since only the periodic microscopic truss unit cells are taken into account.

The periodic BC is illustrated in Fig. 3, where two corresponding microscopic nodes located on two opposite edges of the unit cell are bound by the relative displacements Δu and Δv in x and y directions, respectively. As an example, to construct the multiscale shape function of macroscopic node 1 in x direction, Δu varies linearly from 1 to 0 along the edge 12 (from the macroscopic node 1–2) and the edge 14. For the other two edges, Δu is set as zero. In addition, another relative displacement Δv is set as zero for all the edges. To avoid the rigid body movement of unit cell, the macroscopic node 3 should be fixed in both x and y directions. After imposing these relative displacement constraints, the multiscale shape function of macroscopic node 1 in x direction can be obtained by solving the static equilibrium equation on the unit cell domain. For more details, one can refer to our previous work [25–33].

The above-constructed multiscale shape function vector of the macroscopic node 1 in the x direction could be denoted as $\boldsymbol{N}_{1x}$, which can be further expressed as

$$\boldsymbol{N}_{1x} = \begin{Bmatrix} \boldsymbol{N}_{1xx} \\ \boldsymbol{N}_{1xy} \end{Bmatrix} \tag{1}$$

where $\boldsymbol{N}_{1xx}$ and $\boldsymbol{N}_{1xy}$ are the components of the vector $\boldsymbol{N}_{1x}$ in x and y directions, respectively. In addition, the dimension of $\boldsymbol{N}_{1x}$ is $2n_s \times 1$ with n_s being the number of microscopic nodes within the unit cell domain. For illustration purpose, $\boldsymbol{N}_{1x}$ is plotted in Fig. 4.

Similarly, the numerical multiscale shape functions of other macroscopic nodes can also be obtained by solving the static equilibrium equations on the unit cell domain with corresponding