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Extended multiscale finite element method for small-deflection analysis of thin composite plates with aperiodic microstructure characteristics

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ABSTRACT

Based on the theoretical framework of the extended multiscale finite element method, an efficient multiscale finite element method is developed for small-deflection analysis of thin composite plates with aperiodic microstructure characteristics. First of all, the decoupling displacement boundary conditions for deflections and rotations are reconstructed based on their coupled displacement modes of thin composite plates. Then, the multiscale base functions can be constructed numerically under the boundary conditions through the micro scale computations. Moreover, the coupling effects of composite laminates, especially asymmetric laminates are considered by introducing the additional coupling terms among translations and rotations into the multiscale base functions. Finally, the macroscopic and microscopic response fields are obtained through the macro scale and downscale computation respectively on the basis of multiscale base functions. Numerical examples demonstrate that the developed method possesses high computing accuracy and efficiency compared with the conventional finite element method.

1. Introduction

Fiber-reinforced composite material has been widely used in the field of aerospace due to its high strength-weight, high stiffness-weight and outstanding designability characteristic. It is worth noting that the analysis of composite structures is usually a multiscale problem as illustrated in Fig. 1. The relevant levels of composite structures from low to high are constituent level, lamina or laminate level and structural level [1]. Consider that the properties and responses of composite structures at higher levels are influenced by those at lower levels. In order to utilize and design composite structures better, it is important to implement the multiscale analysis. As shown in Fig. 1, the microscopic characters of composite structures at different levels could be various and aperiodic, such as the distributions of constituents, microscopic holes and hierarchical stiffeners [2–6]. Since the conventional finite element method (FEM) needs mesh refinement, which leads to a huge number of elements and low computing efficiency. Therefore, to find a multiscale numerical computation method with high computing efficiency and required precision has become a research hotspot in recent years.

The multiscale method in the field of fiber-reinforced composite material is mainly the micromechanics method [7,8]. It aims at establishing the relationship between the macroscopic mechanical properties and microstructure characteristics. At present, the micromechanics methods which have been widely applied mainly include the asymptotic homogenization method, the representative volume element method, the multiscale finite element method, etc.

The asymptotic homogenization method has strict mathematical basis, which was first proposed by Bensoussan [9] for the heterogeneous material with periodic structural characteristics. Based on the asymptotic homogenization theory, Guedes [10] proposed the computational homogenization method. In this method, the FEM is applied to obtain the homogenized macroscopic material parameter and the microscopic valuable information quantities [11]. The computational homogenization method has been applied to solve the nonlinear problems of the composites [12–15]. For the heterogeneous material with random microscopic characteristics, the homogenization theory generally needs to select a relatively large representative volume element to contain its microscopic characteristics due to the restrictions of periodicity assumption. That leads to a loss of computing efficiency.

The representative volume element method is a kind of the homogenization method based on numerical values. This method can obtain the equivalent mechanical properties of heterogeneous material as well as the macroscopic and microscopic stress and strain fields by choosing a representative volume element and making analysis after setting specific boundary conditions [16].

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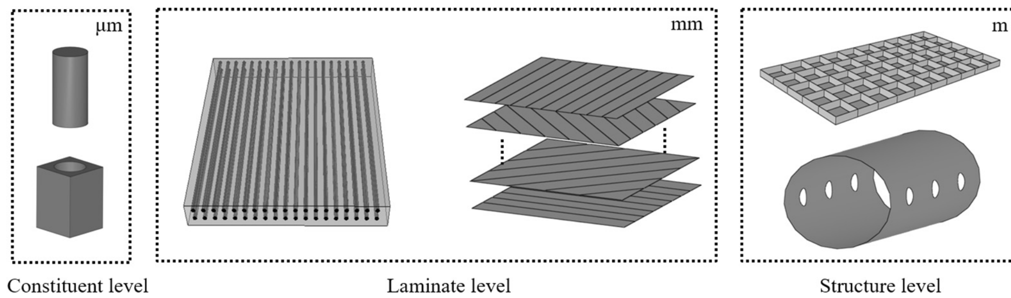

Fig. 1. Illustration of the relevant levels for fiber-reinforced composite structures.

The boundary conditions frequently used in the representative volume element method include Dirichlet boundary condition, Neumann boundary condition and periodic boundary condition. Hazanov [17] compared the effect of different boundary conditions on the accuracy of the representative volume element method and showed that the macroscopic elastic modulus of heterogeneous material tend to upper bound and lower bound respectively under Dirichlet and Neumann boundary conditions. Because the selection of boundary conditions of the representative volume element method is demanding, there is no guarantee that the boundary conditions imposed forcibly consist with actual deformations. Therefore, the predicted results may be distorted. In addition, the selection of representative volume elements depends entirely on the user's experience. Thus, the size and size effect of representative volume elements should be predetermined [18,19].

The basic idea of the multiscale finite element method (MsFEM) can date back to the research work of Babuska [20,21]. Afterwards, Hou et al. [22,23] further developed and officially named it as the multiscale finite element method. This method achieves multiscale analysis by constructing the multiscale base functions which reflect the microstructure characteristics. The construction of the multiscale base functions is different from the conventional finite element base functions. They are obtained by numerically solving the displacement field of macroscopic elements, so the material properties within the macroscopic elements do not have to be uniform. Since the problems can be solved at the macro scale directly on the basis of the multiscale base functions, the computing efficiency is improved. Compared with other multiscale methods, the MsFEM is not restricted by the periodicity assumption and can execute the downscale computation easily. The MsFEM has been used for the flow numerical simulation [24,25]. Moreover, several similar multiscale computational methods have been developed, including the variational multiscale method [26,27], the Voronoi cell method [28,29] the multiscale finite volume method [30,31], generalized multiscale finite element method [32–34], etc. However, for the vector field (the stress and strain fields) problems of composite materials, the methods above are still not applicable. The extended multiscale finite element method (EMsFEM) is developed from the MsFEM by Zhang et al. [35]. In order to consider the coupling effects among different directions for the multi-dimensional vector field problems, the additional coupling terms of the multiscale base functions are introduced. The research [35] shows that the EMsFEM has the advantages of high computing efficiency and accuracy. It has been successfully applied in the research of the vector field problems of heterogeneous porous media and periodic truss material [36–39].

At present, less literature has discussed about the applications of the EMsFEM for the vector filed problems of thin composite plates. The difficulty lies in the following aspects. First, the anisotropy and layup configurations of composite plates increase the complexity of multiscale analysis. Second, the additional terms within the multiscale base functions are only employed to consider the coupling effects among different directions in the present EMsFEM. But for the fiber-reinforced composite laminates, especially asymmetric laminates, the other additional coupling terms need to be introduced to consider the coupling effects among translations and rotations. Last but not least, it is necessary to reconstruct the decoupling displacement boundary conditions for deflections and rotations when constructing the multiscale base functions of thin composite plates because the displacement modes of deflections and rotations are both nonlinear and coupled with each other based on the thin plate theory.

Based on the theoretical framework of the EMsFEM, a novel multiscale finite element method suitable for small-deflection analysis of thin composite plates is developed in this paper. First, the macro scale and micro scale finite element formulations of composite plates with the anisotropy and layup configurations are derived. Then, the decoupling displacement boundary conditions for deflections and rotations are reconstructed by assigning a unit value and zero on the macroscopic nodes and interpolating on the microscopic nodes according to their coupled displacement modes of thin composite plates. Further, the multiscale base functions are constructed numerically under the boundary conditions through the micro scale computations. Consider that the coupling effects of composite laminates, especially asymmetric laminates are strong. The other additional coupling terms among translations and rotations are introduced into the multiscale base functions. Finally, the macroscopic and microscopic response fields are obtained through the macro scale and downscale computation respectively on the basis of multiscale base functions. The numerical examples show that the developed method possesses high computing accuracy for the thin composite plates in consideration of layup configurations and aperiodic microstructure characteristics. In addition, compared with the conventional finite element method, the computing efficiency is improved significantly.

2. Methodology

2.1. Basic principle

The developed method consists of three steps: the micro scale, macro scale and downscale computations. The micro scale computation is employed to construct the multiscale base functions, and then the macroscopic displacement fields can be obtained at the macro scale. The downscale computation is employed to obtain the microscopic response fields, including the strain and stress fields.

The characteristics of the EMsFEM for small-deflection analysis of thin composite plates are as follows. First, the anisotropy and layup configurations of composite plates are considered at the micro scale computation and associated with the macroscopic computation through the multiscale base functions. Second, the