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Pseudo-membrane shell theory of hybrid anisotropic materials

S.W. Chung [a,*], S.G. Hong [b]

[a] School of Architecture, University of Utah, Salt Lake City, UT, USA
[b] Department of Architecture, Seoul National University, Seoul, South Korea

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ABSTRACT

The behavior and characteristics of classical membrane theory of isotropic materials are different from that of anisotropic materials, care must be taken to prevent secondary bending moments due to the unbalanced arrangement of laminates of anisotropic materials. At times, bending theory may have to be adopted and the current design codes, such as ASME, API and ACI must be reviewed for the case of anisotropic materials. The stresses and strains can be significantly different between the pure membrane and bending theories.

This paper derives a membrane type shell theory of hybrid anisotropic materials, governing differential equations together with the procedures to locate the mechanical neutral axis. The theory is derived by first considering generalized stress strain relationship of a three dimensional anisotropic body which is subjected to 21 compliance matrix and then non-dimensionalizing each variable with asymptotic expansion. After applying to the equilibrium and stress-displacement equations, we are allowed to proceed asymptotic integration to reach the first approximation theory. Also possible secondary moments due to the unbalanced built up of lamination are quantifiably expressed. The theory is different from the so called pure membrane or the semi-membrane analysis.

1. Introduction

Shell theories used for pressure vessel design and manufacturing technology are becoming more important recently as the outer space exploration being more active. The pressure vessel ranges from deep water submarines, space vehicles, to the dome type human residences in the Moon or Mars.

The membrane theory of shell is simple and been existing for generations since Trusdell and Goldenveiser have theoretically formulated as shown in the Refs. [7,8].

The mechanics of composites are complicated compared to the ordinary conventional materials such as steel and other metallic brands but composites possess such characteristics as high strength/density and modulus/density ratios, which will allow flight vehicles more efficient and increased distance. The filaments embedded in the matrix materials of composites give additional stiffness and tensile strength. They can be arranged arbitrarily so as to make a structure more resistant to loadings. As the mechanical properties of composites vary depending on the direction of the fiber arrangement, it is necessary to analyze them by use of an anisotropic theory. Also the current design codes including ASME, API and ACI, Refs. [15–18], are all based on membrane theory for isotropic materials.

Pressure vessels of composite materials are, in general, constructed of thin layers of different thickness with different material properties. The properties of anisotropic materials are represented by different elastic coefficients and different cross-ply angles. The cross-ply angle, γ, is the angle between major elastic axis of the material and reference axis (Figs. 1 and 2). The variation in properties in the direction of the thickness implies non-homogeneity of the material and composite structures must thus be analyzed according to theories which allow for non-homogeneous anisotropic material behavior. Our task is to formulate a theory for a shell of composite materials which are non-homogeneous and anisotropic materials.

According to the exact three-dimensional theory of elasticity, a shell element is considered as a volume element. All possible stresses and strains are assumed to exist and no simplifying assumptions are allowed in the formulation of the theory. We therefore allow for six stress components, six strain components and three displacements as indicated in the following equation:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \quad i,j = 1,2,3 \quad k,l = 1,2 \tag{1}$$

where σ_{ij} and ε_{kl} are stress and strain tensors respectively and C_{ijkl} are elastic moduli.

* Corresponding author.
E-mail address: samuelchung00@gmail.com (S.W. Chung).

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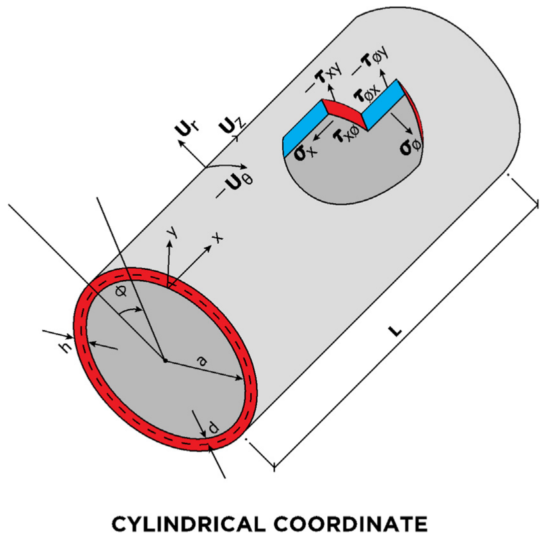

Fig. 1. Dimensions, deformations and stresses of the cylindrical shell.

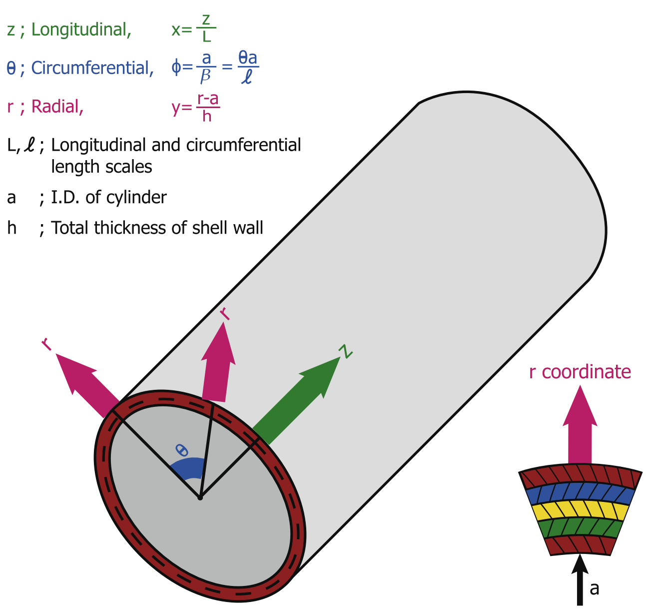

Fig. 2. Details of the coordinate system.

There are thus a total of fifteen unknowns to solve for in a three dimensional elasticity problem. On the other hand, the equilibrium equations and strain displacement equations can be obtained for a volume element and six generalized elasticity equations can be used. A total of fifteen equations can thus be formulated and it is basically possible to set up a solution for a three-dimensional elasticity problem. It is however very complicated to obtain a unique solution which satisfies both the above fifteen equations and the associated boundary conditions. This led to the development of various theories for structures of engineering interest. A detailed description of classical shell theory can be found in various Ref. [1–12].

In the first part of this article, the asymptotic expansion and integration method is used to reduce the exact three-dimensional elasticity theory for a non-homogeneous, anisotropic cylindrical shell to approximate theories. The analysis is made such that it is valid for materials which are non-homogeneous to the extent that their mechanical properties are allowed to vary with the thickness coordinate. The derivation of the theories is accomplished by first introducing the shell dimensions and as yet unspecified characteristic length scales via changes in the independent variables. Next, the dimensionless stresses and displacements are expanded asymptotically by using the thinness of the shell as the expansion parameter. A choice of characteristic length scales is then made and corresponding to different combination of these length scales, different sequences of systems of differential equations are obtained. Subsequent integration over the thickness and satisfaction of the boundary conditions yields the desired equations governing the formulation of the first approximation stress states of non-homogeneous anisotropic cylindrical shell.

2. Formulation of cylindrical shell theory of anisotropic materials

Consider a non-homogeneous, anisotropic volume element of a cylindrical body with longitudinal, circumferential (angular) and radial coordinates being noted as z, θ , r, respectively and subjected to all possible stresses and strains (Fig. 1). The cylinder occupies the space between $a \leqslant r \leqslant a + h$ and the edges are located at $z = 0$ and $z = L$. Here, a is the inner radius, h the thickness and L the length see Table 1.

Assuming that the deformations are sufficiently small so that linear elasticity theory is valid, the following equations govern the problem:

$$\begin{aligned}
&(r\tau_{rz})_{,r} + \tau_{\theta z,\theta} + (r\sigma_z)_{,z} = 0\\
&(r\tau_{r\theta})_{,r} + \sigma_{\theta,\theta} + (r\tau_{\theta z})_{,z} + \tau_{\theta z} = 0\\
&(r\sigma_r)_{,r} + \tau_{r\theta,\theta} + (r\tau_{rz})_{,z} - \sigma_\theta = 0
\end{aligned} \tag{2}$$

$$\begin{aligned}
&u_{z,z} = S_{11}\sigma_z + S_{12}\sigma_\theta + S_{13}\sigma_r + S_{14}\tau_{r\theta} + S_{15}\tau_{rz} + S_{16}\tau_{\theta z}\\
&\tfrac{1}{r}(u_{\theta,\theta} + u_r) = S_{12}\sigma_z + S_{22}\sigma_\theta + \cdots + S_{26}\tau_{\theta z}\\
&u_{r,z} = S_{13}\sigma_z + \cdots + S_{36}\tau_{\theta z}\\
&\tfrac{1}{r}u_{r,\theta} + u_{\theta z} - \tfrac{1}{r}u_\theta = S_{14}\sigma_z + \cdots + S_{16}\tau_{\theta z}\\
&u_{z,r} + u_{r,z} = S_{15}\sigma_z + \cdots + S_{36}\tau_{\theta z}\\
&u_{\theta,z} + \tfrac{1}{r}u_{z,\theta} = S_{16}\sigma_z + \cdots + S_{66}\tau_{\theta z}
\end{aligned} \tag{3}$$

In the above Eqs. (2) are equilibrium equations and (3) stress-displacement relations. In that u_r , u_θ , u_z are the displacement components in the radial, circumferential and longitudinal directions, respectively, σ_r, σ_θ, σ_z the normal stress components in the same directions and $\tau_{\theta z}$, τ_{rz}, $\tau_{r\theta}$ are the shear stresses on the θ-z face, r-z face, r-θ face respectively (Fig. 1). A comma indicates partial differentiation with respect to the indicated coordinates. The

Table 1
List of symbols.

List of symbols	
a:	Inside Radius of Cylindrical Shell
h:	Total Thickness of the Shell Wall
S_i:	Radius of Each Layer of Wall (I = 1, 2, 3 — to the number of layer)
L:	Longitudinal Length Scale to be defined, Also Actual Length of the Cylindrical Shell
E_i:	Young's Moduli in I Direction
G_{ij}:	Shear Moduli in i-j Face
S_{ij}:	Compliance Matrix of Materials of Each Layer
r:	Radial Coordinate
l:	Circumferential Length Scale to be defined
Y:	Angle of Fiber Orientation
σ:	Normal Stresses
ε:	Strains Normal
z, θ, r:	Generalized Coordinates in Longitudinal, Circumferential and Radial Directions Respectively
τ:	Shear Stresses
ε_{ij}:	Shear Strains in i-j Face
λ:	Shell Thickness / Inside Radius (h/a)
C_{ij}:	Elastic Moduli in General
X, φ, Y:	Non Dimensional Coordinate System in Longitudinal, Circumferential and Radial Directions Respectively