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Solution methods for two key problems in multiscale asymptotic expansion method

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ABSTRACT

The accuracy of decoupled multiscale asymptotic expansion method (MsAEM) depends completely on the accuracy of influence functions and the accuracy of homogenized displacement derivatives. Unit cell problem must be solved first for obtaining influence functions whose calculational accuracy depends largely on the boundary conditions of the unit cell problem; and homogenized problem must be solved for obtaining homogenized displacement derivatives whose accuracy depends largely on the order of used finite elements and meshes. In this work, as for two-dimensional (2D) periodical composite structures, super unit cell approach is proposed to solve for accurate influence functions, and quasi potential energy functional corresponding to influence functions (called quasi displacements in this paper), is constructed to evaluate the accuracy of influence functions or boundary conditions of unit cell problem, and it follows that clamp boundary condition is not always suitable for solving influence functions of different orders although it is exact for unit cell problem of one-dimensional (1D) rod. Finally, the differential quadrature finite element method is employed to improve the computational accuracy of homogenized displacement derivatives.

1. Introduction

To balance accuracy and efficiency, various multiscale methods have been motivated, such as the mathematical homogenization method (MHM) [1,2], the generalized finite element method (GFEM) [3,4], the multiscale finite element method (MsFEM) [5,6], the heterogeneous multiscale method (HMM) [7,8] and the multiscale eigenelement method (MEM) [9,10], among of which MHM is essential and representative and has been elaborated in many Refs. [11–19]. And the multiscale asymptotic expansion method (MsAEM) is one of the mathematical homogenization methods widely used.

Present authors [20,21] have investigated the accuracy and physical interpretation of MsAEM, and concluded that:

1) The second expansion term is necessary for satisfying accuracy requirements in theoretical sense.
2) MsAEM can be understood as the superposition method of admissible deformation modes independent each other of different orders, and the coefficients of deformation modes are the homogenized displacement derivatives.
3) The total potential energy functional can be viewed as an effective method to evaluate computational accuracy of the obtained variables as displacements.
4) Super unit cell approach and high accurate differential quadrature finite element method were recommended for respectively improving the computational accuracy of influence functions and homogenized displacement derivatives, which are the objectives of present work, and rarely are there relative publications.

The outline of present paper is as follows: the matrix form of MsAEM is presented, the concept of quasi displacements is introduced in Section 2; in Section 3, as for 2D periodical composite structure, super unit cell approach is proposed to solve for accurate influence functions, further in Section 4, the analytical solutions of influence functions are arrived at for periodical composite rod. And the differential quadrature finite element method (DQFEM) is incorporated into MsAEM to improve the computational accuracy of homogenized displacement derivatives in Section 5. Finally, conclusions are drawn in Section 6.

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2. The multiscale asymptotic expansion method

The governing differential equation for two dimensional composite with multiscale or rough coefficients as follows, which is a classical Dirichlet's problem:

$$-\frac{\partial}{\partial x_j}\left(E^{\varepsilon}_{ijmn}(\boldsymbol{x})\frac{1}{2}\left(\frac{\partial u^{\varepsilon}_m}{\partial x_n}+\frac{\partial u^{\varepsilon}_n}{\partial x_m}\right)\right)=f_i(\boldsymbol{x}) \quad \text{in} \quad \Omega\subset R^3 \qquad u^{\varepsilon}(\boldsymbol{x})=0 \quad \text{on} \quad \partial\Omega \tag{1}$$

where E^{ε}_{ijmn} is the fourth order elastic tensor and the indices $i, j, m, n = 1, 2$. The actual displacement u^{ε}_m in asymptotic expansion form with both macro and micro scales is

$$u^{\varepsilon}_m(\boldsymbol{x}) = u^0_m(\boldsymbol{x}) + \varepsilon u^1_m(\boldsymbol{x},\boldsymbol{y}) + \varepsilon^2 u^2_m(\boldsymbol{x},\boldsymbol{y}) + \cdots \tag{2}$$

where the homogenized displacement u^0_m is offered by homogenized model, the perturbed displacement function $u^j_m(\boldsymbol{x},\boldsymbol{y})$ of both scales is periodic in $\boldsymbol{y}$, and the small parameter ε indicates the dimension proportion between unit cell and entire domain.

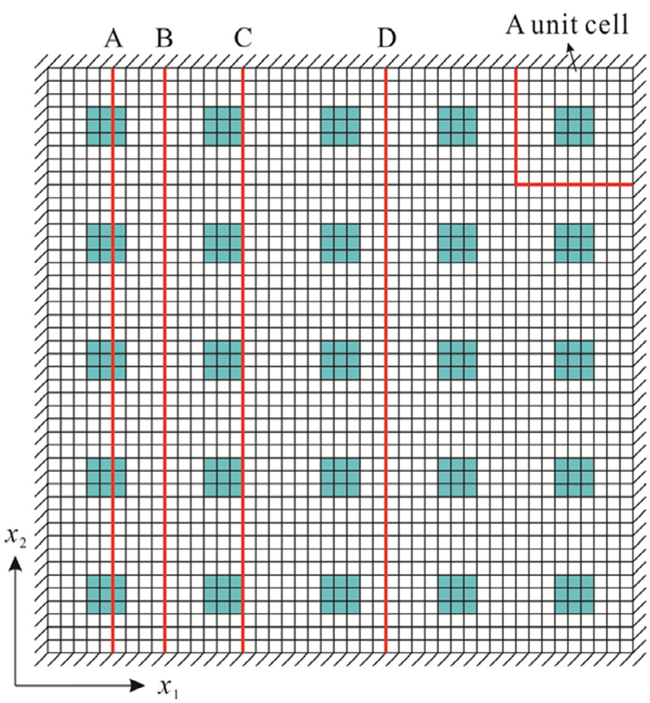


Fig. 1. 2D periodical composite and unit cell with square inclusion.

In reference [20], the necessity of keeping the second order expansion was stressed, therefore the second order expansion term is taken into account in present work. MsAEM up to any expansion order has been formulated in a matrix form [21] which is convenient for use as a standard finite element method (FEM). The governing equation of unit cell problem in discrete form for any order of influence functions is as

$$\boldsymbol{K}^{\varepsilon}\boldsymbol{\chi}_r = \boldsymbol{F}_r \tag{3}$$

where $r = 1, 2, \ldots$ is the expansion order, and $\boldsymbol{K}^{\varepsilon}$ is the stiffness matrix of unit cell micro model, $\boldsymbol{\chi}_r$ and $\boldsymbol{F}_r$ are influence function matrix and quasi load matrix of order r, respectively.

From Eq. (3) one can see that $\boldsymbol{K}^{\varepsilon}$ likes stiffness matrix in FEM, and $\boldsymbol{F}_r$ is quasi load matrix, so hereinafter $\boldsymbol{\chi}_r$ is called quasi displacement corresponding to quasi load $\boldsymbol{F}_r$. If given mesh and element type, the accuracy of $\boldsymbol{\chi}_r$ depends on periodical boundary conditions of unit cell problem, thus the boundary conditions are to be studied below.

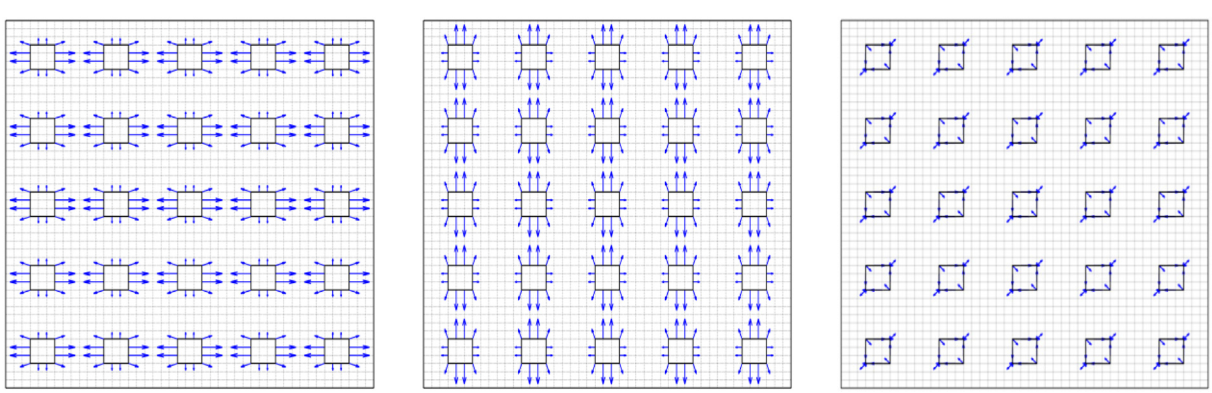
3. Quasi displacements of 2D periodical composite structure

For the clarity of investigation, elastic plane problem of a square 2D periodical composite plate of 45 mm × 45 mm with clamp boundary condition is studied in this section, as shown in Fig. 1; each unit cell is of 9 mm × 9 mm and involves one inclusion. The materials are both isotropic, and Young's moduli $E_1 = 2.97$ GPa (for matrix), $E_2 = 90.585$ GPa (for inclusions), and Poisson's ratio is 0.33.

3.1. Quasi displacements from the micro model of the entire structure

In order to investigate accurate boundary conditions of unit cell problem, the quasi static problem of entire structure subjected to quasi loads is solved first for quasi displacements by using micro mesh of 45 × 45, see Fig. 1, that is the total number of bilinear rectangular element used is 2025. Apparently, the problem is not a real periodical composite one in which the number of unit cells would be much more than present one and it can hardly be solved directly if using micro meshes. Quasi displacements of different orders obtained by this approach (denoted by Approach 1) are taken only as standard to evaluate the results by another two approaches described below.

The first order quasi load $\boldsymbol{F}_1$ is depends only on elastic constants, and three columns of $\boldsymbol{F}_1$ are just like three independent load vectors as in elastic plane problem, as shown in Fig. 2. It follows that $\boldsymbol{F}_1$ has non-zero values along the interface lines between



(a) The 1st column of $\boldsymbol{F}_1$, kl=11 (b) The 2nd column of $\boldsymbol{F}_1$, kl=22 (c) The 3rd column of $\boldsymbol{F}_1$, kl=12

Fig. 2. Diagrams of matrix $\boldsymbol{F}_1$ of the structure.