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Static analysis of bi-directional functionally graded curved beams

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ABSTRACT

Flexure of bi-directional functionally graded (FG) circular beams is analyzed using the kinematical assumptions of the Euler–Bernoulli theory. The material properties are varied along the axis (tangential direction) and thickness (radial direction) of the beam simultaneously. Analytical results are presented for statically-determinate circular cantilever beams under the action of various tip loads. Finally, parametric studies are conducted to investigate the variation of critical stresses and displacements with the gradation parameters. These indicate the possibility of tailoring the response of bi-directional FG beams to fit a wide range of structural constraints.

1. Introduction

Structural efficiency i.e. designing high performance low-weight structures, forms the fundamental design philosophy of the aerospace, automobile, naval and other high performance industries. Hence, thin-walled and slender assemblages of beams, plates and shells which possess high stiffness-to-weight ratios are employed as primary load bearing members in these industries. The advent of modern composite materials technology, particularly in the form of fiber reinforced polymers provides the capability to tailor the performance of structures as per conflicting design requirements [1,2]. Unfortunately, the layered topology of these materials make them susceptible to a wide variety of failure modes at the layer interfaces (delamination, for example) due to the discontinuity in material property variations through the thickness [3]. Functionally graded materials (FGMs) help mitigate these difficulties. FGMs are a novel class of inhomogeneous composites which possess continuous variations of material properties along desired directions and are manufactured by varying the volume fractions of two or more constituents spatially [4]. These advanced materials offer multiple functionalities- for example, by combining the thermal resistance of a ceramic with the toughness, wear resistance and machinability of metals [5–7], whilst the smooth variation of properties helps extenuate the interface problems observed in common composites [8].

Within the purview of functionally graded (FG) beams, there exists a large body of research on the statics, dynamics and stability of straight beams with properties graded only in one direction - either axially [9–12] or through-the-thickness [6,13–21]. The analysis of axially functionally graded beams requires models based on complicated governing equations; these are in the form of differential equations with non-constant coefficients, so emphasis has mostly been laid on the analysis of beams with thickness-wise gradations of material properties. For FG curved beams in particular, a review of literature as detailed below reveals that approximate and elasticity-based models have been developed only for cases where the material properties are varied within the cross-section of the beam.

The in-plane and out-of-plane buckling of doubly-symmetric curved FG beams under thermal loading was studied by Rastgo et al. [22]. By neglecting shear deformation and using a linearized variation of material properties through the thickness of the beam, the stability equations were derived using the principle of virtual work. Buckling studies were also conducted by Shafiee et al. [23] for FG circular arches with a thickness-wise gradation of material properties using variational principles. Closed-form solutions developed for particular loading cases agreed well with known results for isotropic beams. Malakzadeh et al. [24] analysed the out-of-plane free vibrations of FG circular beams in a thermal environment using the differential quadrature method. The governing equations were developed using a first order shear deformation theory taking into account the effects of rotary inertia while the material properties were graded along the beam thickness. Filipich and Piovan [25] studied the dynamics of thick FG curved arches using a beam theory in conjunction with the power series method.

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The material properties were graded along the thickness of the arch and the effect of arch height on the vibration frequency was investigated. Using a first order shear deformation theory along with the Ritz method, Yousefi and Rastgoo [26] analysed the free vibrations of FG spatial curved beams in the form of cylindrical helical springs. The material properties were graded in the direction of curvature of the beam and the effects of pitch angle and number of turns of the helix on the frequency parameter were investigated. Piovan et al. [27] developed a finite element formulation for the in-plane and out-of-plane dynamics and buckling of FG curved beams. A cross-sectional variation of material properties was assumed and shear flexibility, structural damping and warping effects were included. Using the generalized differential quadrature method together with a first order shear deformation theory, Kurtaran [28] analysed the large displacement static and transient behaviour of thick FG circular beams. A power-law variation of material properties through-the-thickness of the beams was considered and the effects of various ceramic–metal material combinations on the displacements were studied. Eroglu [29] analysed the in-plane vibrations of FG circular beams in a thermal environment using a first order shear deformation theory which included the effects of axial deformation and rotary inertia. A power-law variation of material properties was assumed through the depth of the beam and it was found that the simple beam approach was adequate to model such structures. Using a geometrically exact beam theory, Eroglu [30] analysed the large in-plane deflections of planar curved beams made of FGM with the properties graded through-the-thickness. The Variational Iteration Method was employed to solve the governing equations and deflection profiles were determined for half and quarter circle cantilever beams.

Lim et al. [31] developed two-dimensional elasticity solutions for the temperature dependent in-plane vibrations of simply supported FG circular arches using the state space method. The material properties were graded along the thickness direction and the effects of geometrical parameters, temperature and gradient index on the vibration frequency were investigated. Malakzadeh [32] analysed the in-plane free vibrations of thick FG circular arches in a thermal environment using two-dimensional elasticity theory. The differential quadrature method was used to solve the thermoelastic equations and the equations of motion which were developed using Hamilton's principle. Parametric studies were conducted to determine the effects of temperature rise, boundary conditions and material parameters, which were graded along the beam thickness, on the vibration frequencies. Dryden[33] studied the stress distribution in radially graded circular beams subjected to pure bending using a nominally generalized form for the spatial variation of the elastic stiffness and found that the standard curved beam approximation exhibits excellent agreement with the exact results. Furthermore, he also presented a method to tailor the gradation to achieve a specified stress profile. Wang and Liu [34] developed elasticity solutions for multilayered FG orthotropic curved beams using the Airy stress function method. Radial variations in the compliance parameters were considered and stresses and displacements were determined for curved cantilever beams under the action of a uniformly distributed load and a pure moment.

All the models detailed above have been used to analyse the mechanics of FG curved beams with properties graded radially/along the thickness. Depending on design considerations, there may exist practical scenarios which necessitate the gradation of material properties along the axis and thickness of the beam simultaneously. The paucity of such models for curved FG beams provides the motivation for this research. Recently, Wang et al. [35] analysed the free vibrations of two-directional FG straight beams using kinematic assumptions of the Euler–Bernoulli theory. An abrupt jump in natural frequencies was reported for particular gradation parameters indicating the importance of investigating multi-directional FGM structures. In this context, the focus of this work is to analyse the statics of bi-directional functionally graded circular beams. Smooth functional variations of the material properties are assumed along the beam axis and thickness simultaneously. The governing equations are developed using the assumptions of the classical hairbrush hypothesis and are solved analytically for cantilevered circular beams under the action of various tip loads. Finally, parametric studies are conducted to investigate the variation of critical stresses and displacements with the gradation parameters.

2. Mathematical formulation

Consider a slender bi-directional functionally graded circular curved beam with radius R_0 (measured at the centroidal axis) and rectangular cross-section with breadth b and thickness h, subtending an angle of θ_{tip} radians at the center of the polar coordinate frame $(\mathbf{e}_r, \mathbf{e}_\theta)$, as shown in Fig. 1. The distance between an arbitrary point (r, θ) within the beam and the centroidal axis is denoted by a local coordinate y, measured positive in the direction of $\mathbf{e}_r$, so that $y = (r - R_0)$. The distance between the neutral axis and the centroidal axis is denoted by μ , also measured positive in the direction of $\mathbf{e}_r$. The kinematical assumptions of the classical Euler–Bernoulli beam theory viz. plane sections remain plane, undistorted and normal to the deformed beam axis, are used to model the deformation of the beam undergoing planar symmetrical bending.

2.1. Kinematics of deformation

Under the assumptions of the Euler–Bernoulli hypothesis, the motion of any cross-section plane can be decomposed into a planar rigid-body translation of its centroid along with a rotation $\phi(\theta)$ of the plane about an axis perpendicular to the $(\mathbf{e}_r \times \mathbf{e}_\theta)$ plane and passing through the centroid (see Fig. 2). Let the rigid-body translations of the cross-section along $\mathbf{e}_r$ and $\mathbf{e}_\theta$ be $u_r^0(\theta)$ and $u_\theta^0(\theta)$ respectively. With this, the displacement components u_r (along $\mathbf{e}_r$) and u_θ (along $\mathbf{e}_\theta$) of an arbitrary point of the beam may be given as

$$u_r = u_r^0, \quad u_\theta = u_\theta^0 + (r - R_0)\phi \tag{1}$$

The classical 'hairbrush' approximation neglects the effects of the transverse (normal and shear) stresses and strains on the deformation of the beam. Hence, the only strain considered in the theory is the axial (tangential) strain $\varepsilon_\theta(r, \theta)$ given by

$$\varepsilon_\theta = \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} = \frac{1}{r}\left\{u_r^0 + (u_\theta^0)' + (r - R_0)\phi'\right\} \tag{2}$$

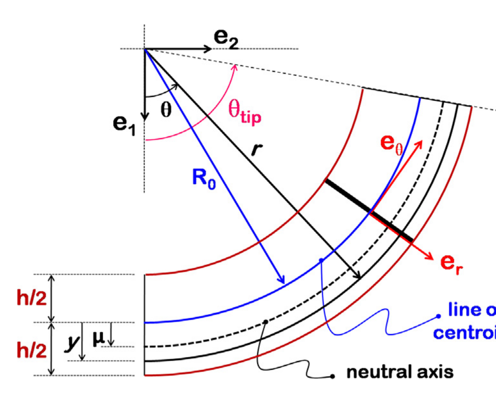

Fig. 1. Geometry of the curved beam.