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Bending analysis of composite laminated and sandwich structures using sublaminate variable-kinematic Ritz models

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ABSTRACT

This paper presents a novel numerical tool for the bending analysis of thin and thick composite plates, including monolithic and sandwich structures. The formulation is developed within a displacement-based approach, where the Principle of Virtual Displacements (PVD) and the method of Ritz are adopted to derive the governing equations. The approach relies upon the Sublaminate Generalized Unified Formulation (S-GUF) as underlying kinematic theory describing the behavior across the plate thickness. Main idea of the S-GUF is to group the plies into a number of smaller units called sublaminates, each of them characterized by an independent, variable-kinematic theory. Continuity conditions between the sublaminates are enforced in strong form during the assembly procedure of the governing equations. The S-GUF appears particularly useful when theories of different order are needed to approximate the displacement field of different portions of the structure, such as in the case of sandwich panels. A number of test cases from the literature is discussed, and results are validated against exact 3D solutions. The results demonstrate the ability of the approach to obtain accurate results, both in terms of deformed shapes, and intra- and inter-laminar stress distributions. A set of novel results is also presented for future benchmarking purposes.

1. Introduction

The ability to accurately predict complex stress distributions, both in terms of normal and transverse stress components, is a crucial aspect to assist the design of modern composite structures. Particularly relevant is the development of analysis tools that can efficiently combine the accuracy of the predictions with a reasonable computational effort, so that the effect of different stacking sequences, material properties or structural configurations, can be assessed since the preliminary design steps.

Several modeling strategies have been developed in the years, including Equivalent Single Layer (ESL) and Layer-Wise (LW) theories [1]. In ESL models, the displacement field is approximated starting from the description of one single reference surface; the number of degrees of freedom is thus independent of the number of layers. However, the in-plane displacement components are approximated with functions that are, at least, C^1 continuous along the thickness, which can be unable to capture the kinematics between layers with drastically different mechanical properties. One approach aiming at overcoming this restriction consists in the adoption of Zig-Zag functions, whose idea is to enrich the kinematics of ESL models by means of shape functions with discontinuous slope at the interface between layers. A comprehensive review of Zig-Zag theories, which is not the subject of the present work, is provided in [2].

Layerwise models are another class of theories directed toward the possibility of capturing complex responses along the through-the-thickness direction. In this case, the displacement field is C^0 continuous along the thickness direction and is described with independent degrees of freedom for each of the plies composing the laminate. It follows that abrupt changes of strains between the layers can be properly detected. However, the number of theory-related degrees of freedom can be high, in particular for multilayered structures composed of a large number of plies.

Observing that the most important variations of elastic properties are generally confined to a subset of layer interfaces – for instance, between the core and the facesheet in the case of a sandwich – an effective idea is to group the plies into smaller sets, sometimes denoted as sublaminates, sharing the same kinematic description. An example is found in [3], where sandwich beams are analyzed using beam theory for the skins and a

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two-dimensional elasticity theory for the core. Refined plate and beam theories where the laminate is represented as an assemblage of sublaminates are discussed in [4–6]. In these formulations, first-order zig-zag kinematics are postulated within each sublaminate. It follows that the in-plane displacement components vary in a piecewise fashion along the thickness, while the normal displacement component varies linearly. The extension to high-order zig-zag kinematics is discussed in [7]. Another approach based on the sublaminate description is found in [8,9], where the displacement field is expanded with a third-order theory, and shear warping and shear coupling functions are used to ensure continuity of in-plane displacements, inter-laminar shear stresses and transverse normal stresses. An idealization of the sandwich into three sublaminates, each of them modeled with FSDT, is proposed in Ref. [10] with regard to the finite element implementation.

While the idea of sublaminate has been adopted by several authors, the formulations in the literature are commonly developed for specific kinematic theories. A theoretical framework that employs a general sublaminate approach in conjunction with general kinematics assumptions has been presented in [11], where a strong form analysis of the governing equations has been employed. The development of a sublaminate model in the context of a variationally consistent variable-kinematic approach has been recently performed by one of the authors [12]: the sublaminate description is coupled with a variable-kinematic theory expressed in a unified formulation, and the governing equations are obtained through the use of displacement-based or mixed variational statements. A vast amount of literature is available regarding unified theories [13,14] and their implementation in the context of numerical procedures based on the finite element method [15–19], radial basis functions [20], quadrature techniques [21,22] and Ritz method [23–25], or of exact solutions based on Navier [26–28] and Levy methods [29–31]. The original idea of unified formulation – due to Carrera and often referred to as CUF (Carrera's Unified Formulation) –, offers the advantage of providing a systematic approach for developing formulations based on ESL and LW theories, of various order, in the context of the same framework. An interesting extension of CUF is due to Demasi [32–37] and is represented by the so-called Generalized Unified Formulation (GUF), the main distinction with CUF being the possibility of expanding each displacement field component by means of different theories and different orders. In the work of Ref. [12], the subdivision of the laminate into sublaminates has been proposed in conjunction with GUF thus leading to the Sublaminate-GUF (S-GUF) approach, and benchmark results are derived from the Navier solutions of the strong-form governing equations. In the present paper, the implementation of the S-GUF approach is discussed in the context of a displacement-based formulation, where the Ritz method is employed as solution technique. The main advantage of the Ritz approach consists in permitting the analysis of any combination of boundary conditions. Furthermore, no restrictions on the stacking sequences exist, so that realistic configurations characterized by the presence of membrane and/or flexural anisotropy can be accounted for. An overview of the S-GUF theoretical framework is provided in Section 2, while a description of the approximate solution approach based on the method of Ritz, together with the assembly and the expansion of the governing equations, is discussed in Section 3. A comprehensive set of test cases is discussed in Section 4, where results from literature are taken for validation purposes and novel benchmark results proposed.

2. The Sublaminate Generalized Unified Formulation

The formulation here presented provides a unified and versatile framework capable of generating multiple-kinematic models of increasing complexity (from classical FSDT models to higher-order full LW models) such that the desired balance of accuracy and computational cost can be obtained for the solution of a wide range of multilayered plate problems. This goal is achieved through the concept of selective ply grouping, or sublaminate, and the variable-kinematic capabilities of the generalized unified formulation (GUF) [32,33]. For this reason, the theoretical framework here proposed will be denoted as Sublaminate Generalized Unified Formulation (S-GUF). The fundamental element of S-GUF is the sublaminate, which is defined as a specific group of adjacent material plies with a specific 2D kinematic description, i.e., the theory adopted to approximate the displacement field across the thickness of the sublaminate. Accordingly, each sublaminate is associated with the following parameters: the number of plies of the sublaminate, the first and last ply constituting the sublaminate, and the local kinematic description, i.e. the Equivalent Single Layer (ESL) or Layerwise (LW) model to approximate the displacement field within the sublaminate. It is important to remark that plate descriptions combining both ESL and LW theories can be accounted for. For instance, a group of plies belonging to a sublaminate could be represented with a ESL description, while those belonging to another sublaminate could be modeled in a LW manner. Similarly, the order of the theory can be chosen independently from sublaminate to sublaminate. One example could be represented by a sandwich panel, whose facesheets are modeled with a low-order ESL theory, while a higher-order theory is adopted for the core. When the laminate is modeled by using one single sublaminate, the classical ESL and LW models are directly recovered.

2.1. Geometric description

The idealization of the multilayered structure as an assembly of perfectly bonded physical plies and mathematical sublaminates is illustrated in Fig. 1. As seen, the laminate is composed of N_p plies of homogeneous, orthotropic material, that are numbered from the bottom to the top of the panel. The thickness of each single ply is denoted as h_p, so that the total thickness of the laminate is $h = \sum_{p=1}^{N_p} h_p$. Following the S-GUF approach, the laminate is subdivided into $k = 1, 2, \ldots, N_k$ sublaminates, numbered from the bottom to the top, each of them characterized by thickness h_k.

The number of plies of the kth sublaminate is denoted as N_p^k, thus $\sum_{k=1}^{N_k} N_p^k = N_p$. A local numbering of plies $p = 1, \ldots, N_p^k$ is also introduced at sublaminate level (see Fig. 2), where the first ply of the sublaminate is $p = 1$, and the last ply is N_p^k. Accordingly, all the relevant quantities belonging to ply p of sublaminate k will be, hereinafter, explicitly indicated with the superscript $()^{p,k}$.

Note that $z \in [-h/2, h/2]$ defines the global thickness coordinate, whereas $z_p \in [-h_p/2, h_p/2]$ and $z_k \in [-h_k/2, h_k/2]$ are the local ply and sublaminate coordinates, respectively. Corresponding nondimensional coordinates are introduced as

$$\zeta_p = \frac{z_p}{h_p/2} \quad \text{and} \quad \zeta_k = \frac{z_k}{h_k/2} \tag{1}$$

and are linked through the following relation:

$$\zeta_p = \frac{h_k}{h_p}\zeta_k + \frac{2}{h_p}\left(z_{0k} - z_{0p}\right) \tag{2}$$

where z_{0p} and z_{0k} are the midplane coordinates of the pth ply and kth laminate, respectively.

2.2. Kinematic approximation at sublaminate level

In the context of the S-GUF formulation, each sublaminate is associated with a specific kinematic assumption that is defined