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Exploiting the benefits of multi-scale analysis in reliability analysis for composite structures

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ABSTRACT

This paper investigates two critical issues, namely propagation of multi-scale uncertainty, and selection of failure criteria, related to reliability analysis of composites by using multi-scale methods. Due to the multi-scale architecture of composites, uncertainties exist in both microscale and macroscale parameters. It is necessary, therefore, to consider random variables at various length scales to ensure accurate estimates of the reliability of composites. Three types of homogenization methods, namely rule of mixtures, Mori–Tanaka and computational homogenization, are adopted to link these two scales, and to propagate uncertainty from micro to macro scales. By integrating these homogenization methods with the stochastic finite element method and structural reliability methods, the reliability of composites can be investigated with a limit state function based on a chosen failure criterion. This multi-scale reliability analysis procedure has been applied to analyse laminated fibre reinforced composites made of AS4/3501 carbon/epoxy. Firstly, a comparative study has been conducted to evaluate the performance of the assumed homogenization methods for the reliability of composites, and to identify advantages compared with a single scale analysis. The results show that multi-scale analysis can provide more accurate reliability estimates. Secondly, several popularly used failure criteria for composites have been compared using multi-scale reliability analysis.

1. Introduction

The trend towards the increasing use of composites is being seen in diverse industries including aerospace, automotive, marine and construction. Advances in design and structural analysis approaches are eagerly required to fully exploit the benefits brought by these materials as other new construction materials [1,2]. Typical fibre reinforced polymer composite structures are made up of laminates which in turn are obtained by stacking individual plies with different fibre orientation. This leads to three different entities including ply, laminate, and component, whose mechanical behaviour is characterised by three different scales, namely fibre diameter, ply and laminate thickness, respectively. Design performance is often related to the probability of failure and requires an understanding of the interaction of uncertain characteristics at both material and structure levels. It is for these reasons that a formal probabilistic analysis is proposed to study the performance of composite materials, components, and structures.

Probability-based methods are powerful tools to design structures under uncertainty. In structural engineering, reliability is one of the most used indicators that interprets response information for design, maintenance, repair, etc. In recent decades, a large number of articles have contributed to the understanding of the probabilistic failure and reliability of composites. For example, [3–5]. However, most of the previous research consider uncertainties at ply- or/and structural-level parameters only, while uncertainties at micro-level parameters are ignored. Owing to the multi-scale architecture of composite materials, variability in structural responses is affected by the variations in parameters at various length scales. As has long been recognised, laminate stress analysis and lamina failure criterion are two critical elements in failure analysis of laminated composite structures. Multi-scale modelling methods are ideal tools to link micro-scale parameters with macro-scale parameters and to propagate uncertainties from micro-scale to macro-scale [6,7]. They have demonstrated their capability to provide sufficiently accurate structural performance simulations due to the fact that it does not require any assumption on the constitutive model at ply level. For instance, unidirectional

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fibre reinforced composites are commonly assumed to be transversely isotropic materials. Consequently, estimation of reliability can be improved using multi-scale method [8]. Several researchers have attempted to conduct multi-scale reliability analyses that consider uncertainties at both micro-scale and macro-scale [9–12]. Concerning the second issue, despite years of extensive research around the world, a complete and validated methodology for predicting the behaviour of composite structures including the effects of damage has not yet been fully achieved with the exception of a few recent research contributions [13,14]. A brief review of failure criteria for composites is provided in the next section. Although qualitative evaluations [15–17], quantitatively experimental comparisons [18], and numerical comparisons [19] have been made for deterministic failure criteria, quantitative comparisons of their performance considering uncertainty are much less prevalent [20].

The objective of this paper is to exploit benefits provided by the adoption of multi-scale analysis for the reliability analysis of composites. Different homogenization methods including rule of mixtures, Mori–Tanaka and computational homogenization have been adopted to link micro-scale parameters with macro-scale parameters and propagated uncertainties from micro to macro. These methods have then been integrated with stochastic finite element method and structural reliability method to conduct reliability analysis for composites. Benefits of multi-scale analysis have been investigated by a series of comparative studies on the effective elastic properties and their statistics, and reliability estimates. In addition, a comparative study of some frequently used failure criteria has been performed from a structural reliability analysis perspective. This has been conducted by using computational homogenization method based multi-scale reliability analysis. Using this approach for the reliability analysis enables us to investigate the influences of variations in micro-scale and macro-scale parameters on the reliability estimation, and to quantify the relative importance of various uncertainties. Consequently, the sources of differences among various failure criteria can be identified in a new and unique way. In comparison with a deterministic failure analysis, a reliability analysis requires to conduct multiple structural analyses. For instance, several iterations are required to obtain a sufficiently accurate estimate of reliability when using the first order reliability method. In practice, finite element methods have become standard tools for numerical simulation of structural response. Hence, failure criteria incorporated in a finite element-based reliability analysis should be efficient in terms of implementation. In accordance with these overarching considerations, six failure criteria were finally selected in the present study: (1) maximum stress, (2) Hashin failure theory, (3) Tsai–Hill failure theory, (4) Tsai–Wu theory, (5) Christensen failure theory, and (6) Hoffman failure theory. Numerical analyses for a lamina and a quasi-isotropic laminate $(0^\circ/\pm 45^\circ/90^\circ)_s$ made of AS4/3501 carbon/epoxy were adopted for the comparative studies.

2. Brief literature review on failure criteria for unidirectional fibre reinforced polymer composites

Composite materials display a wide variety of failure mechanisms including fibre failure, matrix cracking, buckling at several scales, and delamination as a result of their complex structural behaviour. Hence, various limit state functions (LSF) can be established depending on the specific problem under consideration. In this study, we limit our focus to laminate failure under in-plane loading conditions, which are largely based on the stress components of an individual ply within the laminate. Here, a brief review of theories for in-plane failure is presented. Readers are referred to find more comprehensive reviews, such as the state-of-the art composite failure theories are included in [17]. Generally, failure criteria can be broadly classified into two groups according to whether failure modes are separated or not.

2.1. Failure theories without failure modes

Firstly, we focus on failure criteria where the ply failure modes are not considered, but the failure of the entire ply is predicted. From the application point of view, this group is easier and more convenient to use, especially in reliability analysis which involves iterative calculations. However, they are often criticised due to their lack of phenomenological basis. This group includes criteria from papers in which the difference between fibre and matrix failure is either unclear or not specified and the so-called fully interactive criteria. Hence, a single quadratic or higher order polynomial equation containing all stress (or strain) components is used to predict the failure. Tsai–Hill failure theory [21] is one of the criteria in this group that modifies the Hill's anisotropic failure theory derived from the Von Mises yield criterion for metals [22]. To account for different strengths in tension and compression, Hoffman added linear terms to Hill's equation, defining the Hoffman failure criterion [23]. Tsai and Wu further developed these criteria to improve their performance in the representation of experimental data that results in the well-known curve fitting based Tsai–Wu failure criterion [24]. As one of the 19 leading failure criteria used in the worldwide failure exercises, Tsai–Wu failure criterion presented in [25,26] shows good performance [18].

2.2. Failure theories with consideration of failure modes

Hashin and Rotem [27], for the first time, proposed failure of laminated composites attributed to different physical phenomena including fibre controlled and matrix controlled failure modes. Many failure criteria have been developed following this idea, and new failure mode based criteria, which are also called physically-based failure criteria in the literature, are increasingly being proposed. We only mentioned few of them in this brief review.

2.2.1. Fibre failure

Fibre failure occurs due to the accumulation of individual fibre failures within plies, which become critical when there are insufficient intact fibres remaining to carry the required loads. For fibre tensile failure, it is generally acknowledged that the maximum stress or maximum strain theory is sufficient to predict this failure mechanism, such as stated in [28]. To consider the interaction between different stress components, more sophisticated models, have been developed. Hashin [29] used a quadratic interaction criterion involving in-plane shear. Chang and Chang [30] improved Hashin's criterion [29] by incorporating nonlinear shear behaviour. Puck and Schürmann [31] modified the maximum strain criterion to include the transverse normal stress through a stress magnification factor. Research in fibre compressive failure is still being performed as the failure mechanism is complex. Depending on the materials, microbuckling and kinking are often recognised as the main failure mode under compressive loading in longitudinal direction [32]. A number of approaches have been developed for incorporating the effects of microbuckling and kinking, e.g. [31,33,34].

2.2.2. Matrix failure

Due to the unique feature of fibre-reinforced composites offering relatively lower properties in the transverse direction compared with the high strength and stiffness properties in the longitudinal direction, matrix cracks are usually the first observed form of damage in fibre reinforced composites [35]. The matrix microcracks usually initiate at defects or fibre–matrix interfaces,