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Research Paper

A one-dimensional integral approach to calculating the failure probability of geotechnical engineering structures

Yonghua Su [a,*], Yanbing Fang [a], Shuai Li [a], Ya Su [a], Xiang Li [b]

[a] College of Civil Engineering, Hunan University, Changsha, Hunan 410082, China
[b] School of Resources and Safety Engineering, Central South University, Changsha, Hunan 410083, China

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ABSTRACT

This study focuses on the geotechnical engineering structures with implicit or unknown expressions of performance functions. A one-dimensional integral approach (ODIA) consisting of sampling, evaluation of statistical moments for multivariable functions, probability density function fitting, and simple integration of failure probability was developed through system integration. A convergence study of an illustrative example was conducted, and the error analysis revealed that the accuracy of ODIA is equivalent to that of the second-order reliability method. Applications of ODIA to a slope and surrounding rock of an excavation were presented to further confirm the accuracy, efficiency, and practicability of the approach.

1. Introduction

In the stability analysis and design of geotechnical engineering structures, the probability of failure is increasingly evaluated [1–4] to rationally consider unavoidable uncertainties in the mechanical properties of different types of geomaterials, geometric shapes of engineering structures, etc.

Assume that a random vector $\mathbf{X} = (X_1, X_2, \ldots, X_n)$ involves n stochastic variables for a geotechnical engineering structure, whose corresponding performance function is $Z = g(\mathbf{X})$. According to the original concept of reliability, an accurate solution to failure probability can be calculated using one of the following integrals:

$$P_f = P[g(\mathbf{X}) \leqslant 0] = \int_{\Omega} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x}, \tag{1}$$

$$P_f = P[Z \leqslant 0] = \int_{\Omega^*} f_Z(z)dz, \tag{2}$$

where P_f is the failure probability of the structure, $f_{\mathbf{X}}(\mathbf{x})$ is the joint probability density function (JPDF) of $\mathbf{X}$, and Ω is its integral range, which satisfies $g(\mathbf{X}) \leqslant 0$, $f_Z(z)$ is the probability density function (PDF) of the random quantity Z, and Ω^* is its integral range, which satisfies $Z \leqslant 0$.

Eq. (1) has substantially received more attention than Eq. (2) for a long time. However, for common engineering structures, a specific expression of $f_{\mathbf{X}}(\mathbf{x})$ is unavailable. Moreover, even though $f_{\mathbf{X}}(\mathbf{x})$ exists, it is often burdensome to calculate the multidimensional integral immediately. That is, it is usually difficult to calculate P_f immediately by using the right side of Eq. (1). To overcome this difficulty, various approximation methods have been developed. Thus far, the methods may be generally divided into three types. The first type is direct simulation, which includes the Monte Carlo simulation (MCS) method [5,6] and its various improved editions [7,8]. This method requires thousands of calculations of the performance function and is often used to verify the accuracy level of a new approach [9]. The second type is the statistical moment method, mainly the first-order reliability method (FORM) [10–12] and the second-order reliability method (SORM) [13,14]. The third type is the combination of the above two methods such as the updated FORM and SORM, which are obtained by integrating the original FORM and SORM with importance sampling method [15,16].

Among these three methods, the statistical moment method has been widely used because of its concise operational process and acceptable accuracy. However, because the gradient vector of the performance function is calculated using the Newton–Leibniz formula in the statistical moment method, the explicit expression of $g(\mathbf{X})$ must be available. However, for most geotechnical engineering structures, especially for slopes and surrounding rocks of tunnels, the expressions of $g(\mathbf{X})$ may be implicit or unknown. Under these circumstances, to calculate the gradient vector, scholars have

* Corresponding author.
E-mail address: yong_su1965@126.com (Y. Su).

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developed various approximation methods. These methods can generally be divided into two types.

First is the response surface method (RSM) that constructs a substitute for $g(\mathbf{X})$. In the early 1990s [17], a quadratic polynomial was used as the surrogate for $g(\mathbf{X})$ in the RSM. Subsequently, numerous improved practices were attempted. For example, Proppe [18] developed an adaptive local approximation scheme; Tan et al. [19] proposed some surrogate approaches based on radial basis function networks and support vector machines; Zhang et al. [20] suggested a fitting method based on the kriging model; Cheng et al. [21] introduced a substitute means based on an artificial neural network. In addition, the RSM was applied in the reliability analysis of tunnel engineering [22,23]. However, Proppe [18] and Guan et al. [24] noted that the high quality of the substitute in the RSM (including the classic RSM and the improved RSM) could not be guaranteed theoretically.

The second approximation method calculates the gradient vector by using the finite difference technique such as the recursive algorithm FORM approach expanding the practicability of the FORM [25,26]. However, the accuracy of the gradient is closely associated with the selected value of the step length coefficient. Herein, the determination of a value satisfying the accurate requirement is a difficult problem, which is still not completely solved. Moreover, the algorithm underlying the approach is still the Hasofer–Lind–Rackwits–Fiessler (HLRF). Under certain conditions, the convergence rate of the algorithm may be unacceptably slow, and it may even diverge and need to resort to some abstruse optimization theories [27–29].

The aforementioned researches mainly focused on how to cater the requirements of the statistical moment methods. Unlike the above practices, aiming at the reliability problem in geotechnical engineering structures, whose expressions of performance functions are implicit or unknown, there are two objectives in the study. The first is to develop a reliability analysis approach that does not require the calculation of gradient vector. The second is to make the approach appealing and practical for geotechnical practitioners. Therefore, starting from the concept implied in Eq. (2), an evaluation tool of statistical moments for random quantity and a technique to conjecture its PDF were developed. Subsequently, on the basis of hybridizing the tool with the technique, a practical approach, namely one-dimensional integral approach (ODIA), to calculating probability of failure by a simple integral was developed. ODIA is simple enough for geotechnical practitioners to understand and implement.

The subsequent sections of the paper are structured as follows. First, the proposed ODIA and the corresponding operation procedure are described in detail in Section 2. In Section 3, a numerical example is illustrated in detail, and comparison and convergence studies are presented to validate the proposed approach. The applications of the proposed approach to geotechnical engineering structures are demonstrated in Section 4. Section 5 provides the conclusions of this study.

2. The proposed approach

2.1. Transformation and sampling

For stochastic properties of $\mathbf{X}$ and its component X_i $(i = 1, 2, \ldots, n)$, a correlation coefficient matrix $\boldsymbol{\rho} = (\rho_{ij})_{n\times n}$ $(i, j = 1, 2, \ldots, n)$ of $\mathbf{X}$, a mean value μ_{Xi} of X_i, a standard deviation σ_{Xi}, a marginal PDF $f_i(x_i)$, and a marginal cumulative distribution function (CDF) $F_i(x_i)$ are available. On the basis of these statistical data of $\mathbf{X}$ and according to the principle of equal probability, the following marginal transformation [30] may be used to map $\mathbf{X}$ into a standard normal vector $\mathbf{Y} = (Y_1, Y_2, \ldots, Y_n)$.

$$F_i(x_i) = \Phi(y_i), \tag{3}$$

$$y_i = \Phi^{-1}(F_i(x_i)). \tag{4}$$

After mapping $\mathbf{X}$ into the standard normal vector $\mathbf{Y}$, the correlation coefficient matrix, $\boldsymbol{\rho}_0 = (\rho_{0ij})_{n\times n}$, $(i, j = 1, 2, \ldots, n)$ of $\mathbf{Y}$ is different from $\boldsymbol{\rho}$. According to the classical Nataf transformation theory [31,32], the following relationship holds between ρ_{0ij} and ρ_{ij}:

$$\rho_{ij} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}((F_i^{-1}(\Phi(y_i) - \mu_{Xi}))/\sigma_{Xi})((F_j^{-1}(\Phi(y_j) - \mu_{Xj}))/\sigma_{Xj})\phi_2(y_i, y_j, \rho_{0ij})\mathrm{d}y_i\mathrm{d}y_j, \tag{5}$$

where $\Phi(\cdot)$ is a CDF and $\phi_2(y_i, y_j, \rho_{0ij})$ is a joint PDF for y_i and y_j.

For convenience, an excellent approximate approach proposed by Der Kiureghian and Liu [32] is used to avoid the above complication. Let R_{ij} be the ratio of ρ_{0ij} to ρ_{ij}, i.e.,

$$R_{ij} = \rho_{0ij}/\rho_{ij}. \tag{6}$$

As testified by Der Kiureghian and Liu [32], R_{ij} is a function of v_{X_i}, v_{X_j}, and ρ_{ij}, i.e.,

$$R_{ij} = f(v_{X_i}, v_{X_j}, \rho_{ij}), \tag{7}$$

where v_{X_i} and v_{X_j} are the coefficients of variation (CV) of X_i and X_j, respectively. The specific expression for the right side of Eq. (7) is associated with the distribution types of X_i and X_j, and various expressions for different distribution types are well documented by Der Kiureghian and Liu [32] and Ditleven and Madsen [30]. Furthermore, Der Kiureghian and Liu [32] verified that the relative errors between results calculated by Eq. (7) and the corresponding exact values are less than 4%.

Thus, $\boldsymbol{\rho}_0$ is available with each component of ρ_{0ij} determined by Eqs. (6) and (7). As a symmetric matrix, $\boldsymbol{\rho}_0$ can be split into a lower triangular matrix $\boldsymbol{\Gamma}$ and its transpose $\boldsymbol{\Gamma}^{\mathrm{T}}$ by the Cholesky factorization, namely,

$$\boldsymbol{\rho}_0 = \boldsymbol{\Gamma}\boldsymbol{\Gamma}^{\mathrm{T}}. \tag{8}$$

Subsequently, by using the inverse $\boldsymbol{\Gamma}^{-1}$, $\mathbf{Y}$ can be mapped into a mutually independent standard normal vector $\mathbf{U} = (U_1, U_2, \ldots, U_n)$ from Eq. (9) provided below:

$$\mathbf{U}^{\mathrm{T}} = \boldsymbol{\Gamma}^{-1}\mathbf{Y}^{\mathrm{T}}. \tag{9}$$

Moreover, $\boldsymbol{\Gamma}^{-1}$ can also be expressed by its rows as follows:

$$\boldsymbol{\Gamma}^{-1} = (\boldsymbol{\Gamma}'_1 \quad \boldsymbol{\Gamma}'_2 \cdots \boldsymbol{\Gamma}'_n)^{\mathrm{T}}, \tag{10}$$

where $\boldsymbol{\Gamma}'_i = (\tilde{\Gamma}_{i1} \quad \tilde{\Gamma}_{i2} \cdots \tilde{\Gamma}_{in})$ and $\tilde{\Gamma}_{ij}(i, j = 1, 2, \ldots, n)$is a component of $\mathbf{U}$. Assuming that $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is a realization of $\mathbf{X}$ and combining Eq. (4) with Eq. (9), each component u_i of $\mathbf{u}$ (a realization of $\mathbf{U}$) corresponding to $\mathbf{x}$ can be obtained as follows:

$$u_i = \boldsymbol{\Gamma}'_i(\Phi^{-1}(F_1(x_1)), \ldots, \Phi^{-1}(F_n(x_n)))^{\mathrm{T}}. \tag{11}$$

Thus far, the transformation from $\mathbf{X}$ to $\mathbf{U}$ is established completely.

From Eq. (9), the following transformation may be given:

$$\mathbf{Y}^{\mathrm{T}} = \boldsymbol{\Gamma}\mathbf{U}^{\mathrm{T}}. \tag{12}$$

Simultaneously, $\boldsymbol{\Gamma}$ can also be expressed by its rows as follows:

$$\boldsymbol{\Gamma} = (\boldsymbol{\Gamma}''_1 \quad \boldsymbol{\Gamma}''_2 \cdots \boldsymbol{\Gamma}''_n)^{\mathrm{T}}, \tag{13}$$

where $\boldsymbol{\Gamma}''_l = (\Gamma_{l1} \quad \Gamma_{l2} \cdots \Gamma_{ln})$ and $\Gamma_{lm}(l, m = 1, 2, \ldots, n)$ is a component of $\boldsymbol{\Gamma}$. Assuming that $\mathbf{u} = (u_1, u_2, \ldots, u_n)$ is a realization of $\mathbf{U}$ and from Eqs. (3) and (12), each component x_i of $\mathbf{x}$ (a realization of $\mathbf{X}$) corresponding to $\mathbf{u}$ can be calculated as follows: