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Research Paper

# A rigorous solution for the stability of polyhedral rock blocks

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ABSTRACT

Block theory has been widely applied to stability analysis of rock engineering due to its clear concept and elegant geometrical theory. For a general block with multiple discontinuity planes, it is assumed that contact is only maintained on a single plane (single-plane sliding) or two intersecting planes (double-plane sliding) in block theory analysis. Since the normal forces and shear resistances acting on the other discontinuity planes are omitted, it can cause unreasonable estimations of block failure modes and incorrect calculation of factors of safety. In this study, a new method is presented that permits to consider the contribution of the normal forces and shear resistances acting on each discontinuity plane to the block stability. The proposed method meets all of the force-equilibrium and moment-equilibrium conditions and provides a rigorous solution for stability of general blocks with any number of faces and any shape. Some typical polyhedral blocks in rock slopes are analyzed using block theory and the proposed method. The results indicate that the traditional block theory may give a misleading conclusion for the predictions of stability and sliding direction of rock blocks when contact occurs on more than two discontinuity planes.



## 1. Introduction

In a rock slope, there are usually numerous discontinuities of various attitudes and different scales. The spatial intersections of the discontinuities at the free surfaces of the slope may form a series of removable rock blocks with different shapes and sizes. It is necessary to evaluate the stability of these removable blocks to determine the excavation and reinforcement design of the slope.

At present, stability analyses of a tetrahedral rock wedge formed by two intersecting discontinuity planes at the crest of a slope are well established in geotechnical literature. The early contributions to the issue arise from the work of Wittke [1], John [2], Londe et al. [3], Chan and Einstein [4], Hoek and Bray [5], and Priest [6]. Recent developments are given by Kumsar et al. [7], Wang and Yin [8], Yeung et al. [9], Chen [10], Jimenez and Sitar [11], Jiang et al. [12], among others. When a rock slope is intersected by more than two discontinuity sets, polyhedral rock blocks with general shapes can be created. Lin et al. [13], Ikegawa and Hudson [14], Jing [15], Lu [16] and Elmouttie et al. [17] developed geometrical identification methods for polyhedral rock blocks. For this type of complex blocks formed by any number of discontinuity planes, the traditional wedge analysis methods are limited. An alternative method is to use block theory proposed by Goodman and Shi [18]. Block theory is a three-dimensional geometrical method with rigorous mathematical deduction that permits analysis of any shape of rock blocks. By means of the theorem of movability of block theory, one is able to analyze the system of joints and other rock discontinuities to find a list of removable blocks. Once the removable blocks are identified, the failure mode of the blocks can be judged according to kinematical admissibility conditions and the factor of safety of the blocks can be calculated by limit equilibrium methods.

Since block theory was put forward, it has been widely applied to stability analysis and support design of rock engineering due to its clear concept and elegant geometrical theory [19–26]. However, there are limitations to the failure mode and stability analysis of the traditional block theory. Goodman and Shi [18] took the translational failure mode into account only and neglected the effect of the moments of external forces applied to a rock block. To study the rotational failure mechanism, Mauldon and Goodman [27], Tonon [28], and Tonon and Asadollahi [29] extended block theory to include the rotational failure modes and proposed vector analyses of keyblock rotations. Zhao and Wang [30] developed a lower bound limit method that can consider the sliding mode and rotation effect simultaneously. However, these analyses of incorporating rotational modes are limited to tetrahedral blocks bounded by

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joint planes. For general blocks with multiple discontinuity planes, at most two discontinuity planes are taken into account in stability analysis of block theory. This may lead to incorrect results for stability evaluation of the general blocks. Mauldon and Ureta [31] proposed an energy method for determination of the factor of safety against sliding failure for a special prismatic block with multiple sliding planes that form a cylindrical surface. Zhang et al. [32] presented a limit equilibrium analysis for a general block with more than two joint planes by dividing the block into multiple sub-blocks along the line of intersection of joint planes. In both Zhang's and Mauldon-Ureta approaches, only force-equilibrium conditions can be satisfied. Sun et al. [33] proposed further an optimization model based on limit equilibrium analysis to evaluate the stability of rock blocks with multiple sliding planes, which meets all force and moment equilibrium conditions. Since a symmetric plane, or sliding direction, needs to be pre-determined in the analysis, the method proposed by Sun et al. is only appropriate for polyhedral blocks with a symmetrical geometry. The behavior of polyhedral blocks with arbitrary joint planes can also be simulated by numerical methods based on the discontinuous media models, such as 3d DDA and 3d DEM [34–36]. However, the determination of the safety factor by the numerical methods involves complex interface contact treatment and time-consuming trial and error calculations using the reduction of the strength parameters.

The purpose of this study is to develop a rigorous method for stability assessment of general blocks with any number of faces and any shape. To overcome the limitations described above, a new method is proposed that considers the effect of the normal forces and shear resistances acting on multiple discontinuity planes on the block stability. The proposed method meets all of the force-equilibrium and moment-equilibrium conditions and provides a rigorous solution for stability analysis of general blocks. Finally, some typical polyhedral blocks in rock slopes are analyzed using the block theory and proposed method, and some interesting findings are illustrated by this study.

## 2. Block theory

### 2.1. Kinematic analysis

Suppose that a removable block is formed by multiple discontinuity planes and the face and upper surface of a slope (Fig. 1). Let $\mathbf{n}_i$ be the upward unit normal to discontinuity plane $i$, $\mathbf{v}_i$ be the unit normal vector of discontinuity plane $i$ pointing to the interior of the block, $\mathbf{r}$ represent the active resultant acting on the block, and $\mathbf{s}$ represent the direction of movement of the block. There are usually three types of failure modes for the removable block: lifting, single-plane sliding and double-plane sliding. From Goodman and Shi [18], the kinematic admissibility for the block failure can be described as follows:

#### 2.1.1. Lifting

When a removable block is lifting (Fig. 1a), the movement direction of the block is parallel to the direction of the active resultant, that is,

$$\mathbf{s} = \mathbf{r}/|\mathbf{r}| \tag{1}$$

For the failure mode of lifting, the kinematic conditions for the block instability are

$$\mathbf{s} \cdot \mathbf{v}_l > 0 \quad \text{for all } l \tag{2}$$

#### 2.1.2. Single-plane sliding

When a removable block slides along plane $i$ only (Fig. 1b), the movement direction of the block is parallel to $\mathbf{s}_i$, that is,

$$\mathbf{s} = \mathbf{s}_i = (\mathbf{n}_i \times \mathbf{r}) \times \mathbf{n}_i / |\mathbf{n}_i \times \mathbf{r}| \tag{3}$$

where $\mathbf{s}_i$ is the orthographic projection of $\mathbf{r}$ on plane $i$.

For the mode of single-plane sliding, the kinematic conditions for the block failure are

$$\mathbf{r} \cdot \mathbf{v}_i \leqslant 0 \tag{4}$$

and

$$\mathbf{s} \cdot \mathbf{v}_l \geqslant 0 \quad \text{for all } l,\ l \neq i \tag{5}$$

#### 2.1.3. Double-plane sliding

When a removable block slides along discontinuity planes $i$ and $j$ (Fig. 1c), the movement direction of the block is parallel to the intersection line of the two planes, that is,

$$\mathbf{s} = \mathbf{s}_{ij} = \frac{\mathbf{n}_i \times \mathbf{n}_j}{|\mathbf{n}_i \times \mathbf{n}_j|} sign[(\mathbf{n}_i \times \mathbf{n}_j) \cdot \mathbf{r}] \tag{6}$$

For the mode of double-plane sliding, the kinematic conditions for the block failure are

$$\mathbf{s}_i \cdot \mathbf{v}_j \leqslant 0, \quad \mathbf{s}_j \cdot \mathbf{v}_i \leqslant 0 \tag{7}$$

and

$$\mathbf{s} \cdot \mathbf{v}_l \geqslant 0 \quad \text{for all } l,\ l \neq i \text{ or } j \tag{8}$$

Here, $\mathbf{s}_i$ and $\mathbf{s}_j$ are the orthographic projections of $\mathbf{r}$ on planes $i$ and $j$, respectively.

### 2.2. Factor of safety

If the failure mode of a removable block is identified by the kinematic admissibility, the factor of safety of the block can be calculated using the limit equilibrium method. This method was originally proposed for tetrahedral blocks by Wittke [1], John [2] and Londe et al. [3], and further extended to the blocks with any number of faces by Goodman and Shi [18].

As shown in Fig. 2, Goodman and Shi introduced a fictitious force $-F\mathbf{s}$ to bring a removable block to a limit equilibrium state. The resultant of shear forces acting on joint planes can be written as

$$-T\mathbf{s} = \sum_l - N_l \tan\phi_l \mathbf{s} - F\mathbf{s} \tag{9}$$

where $N_l$ is the normal force acting on the discontinuity plane $l$, and $\phi_l$ is the friction angle of discontinuity plane $l$.

For evaluation of the block stability, the conventional definition of safety factor is introduced to bring the block to a limiting state. Further considering the cohesion of joint planes, the resultant of shear forces on discontinuity planes can be rewritten as

$$-T\mathbf{s} = \sum_l \frac{-1}{F_s}(N_l \tan\phi_l + c_l A_l)\mathbf{s} \tag{10}$$

where $F_s$ is the factor of safety, $c_l$ and $A_l$ are the cohesion and area of discontinuity plane $l$.

Therefore, the force equilibrium equations for the block can be expressed as

$$\mathbf{r} + \sum_l N_l \mathbf{v}_l - T\mathbf{s} = 0 \tag{11}$$

#### 2.2.1. Lifting

In this case, all of the discontinuity planes will open and the normal reaction forces $N_l = 0$ for all the discontinuity planes. The factor of safety of the removable block is defined as zero.