Computers and Geotechnics 86 (2017) 95–107

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Research Paper

Generation of multivariate cross-correlated geotechnical random fields

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ARTICLE INFO

Article history:
Received 16 September 2016
Received in revised form 20 December 2016
Accepted 10 January 2017
Available online 18 January 2017

Keywords:
Anisotropy
Cross correlation
Random field
Slope stability
Reliability analysis
Geotechnical uncertainty

ABSTRACT

A geotechnical problem that involves several spatially correlated parameters can be best described using multivariate cross-correlated random fields. The joint distribution of these random variables cannot be uniquely defined using their marginal distributions and correlation coefficients alone. This paper presents a generic methodology for generating multivariate cross-correlated random fields. The joint distribution is rigorously established using a copula function that describes the dependence structure among the individual variables. The cross-correlated random fields are generated through Cholesky decomposition and conditional sampling based on the joint distribution. The random fields are verified regarding the anisotropic scales of fluctuation and copula parameters.

1. Introduction

Soils are variable over space as they exist in natural states and can be restructured as a result of depositional, post depositional or weathering processes. Such spatial variability must be considered to realistically represent the actual soil conditions. Random field theory was developed to characterize the spatial variability of soil properties with point statistics by Vanmarcke [1] and Fenton and Vanmarcke [2], among others. Jaksa and Fenton [3] provided an excellent summary of variance diagrams for a random field. Random fields have been extensively applied to evaluate the performance of geotechnical structures. For instance, the effects of soil spatial variability on slope performance and soil consolidation were investigated by Huang et al. [4,5], Hicks and Spencer [6], Vanmarcke [7] and Kasama and Whittle [8]; the influences of spatially random soil stiffness on displacements and capacities of shallow and deep foundations were investigated by Soubra and Massih [9], Griffiths et al. [10], and many others; and the spatial variability of in-situ weathered soils were studied by Zhang and Dasaka [11], Dasaka and Zhang [12], Li et al. [13] and Li et al. [14] in terms of random fields. Several techniques to characterize a univariate random field were proposed by Uzielli et al. [15], Jha and Ching [16], Zhu et al. [17], Zhu and Zhang [18,19], Vessia and Pula [20] and many others.

There are two types of correlation structures: (1) autocorrelation that describes the relation between the values of one random variable at two locations in a spatially correlated field, and (2) cross-correlation among several random parameters. Both are involved in a multivariate cross-correlated random field but the latter is the subject of investigation in this paper. In the literature, a dependence structure is often considered by establishing a cross-correlation matrix $\mathbf{M}_c$, each term representing the correlation coefficients between random variables. In this manner, Griffiths et al. [21] considered the cross correlation between cohesion and friction angle modeled as random fields while Wu [22] accounted for the two parameters as random constants; Ching and Phoon [23] constructed a multivariate distribution of the shear strength of clays. To facilitate the generation of cross-correlated random fields, a fast Fourier transform method was used to deal with complex-valued issues involved in cross-spectral densities related to the cross-correlation coefficients [24–26]. Furthermore, series expansion methods such as Karhunen–Loe've expansion and expansion optimal linear estimation methods have gained popularity in simulating cross-correlated random fields by invoking a cross-correlation matrix [27], and have been extended to problems related to the bearing capacity of strip footings [28,29]. Spectral representation has often been recognized as another standard technique initiated by Rice [30] and applied by a number of researchers such as Shinozuka and Deodatis [31], Popescu et al. [32], Chen and Letchford [33], Gao et al. [34] and many others. Again, the cross-correlation among random fields was considered by introducing cross-correlation coefficients only.

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http://dx.doi.org/10.1016/j.compgeo.2017.01.006
0266-352X/

Nomenclature

$C[F(x_1),\ldots,F(x_n);\theta]$	copula function
$c[F(x_1),\ldots,F(x_n);\theta]$	copula density function
c	cohesion of soil
D	measured data
$F(x_n)$	marginal distribution of x_n
FS	factor of safety
$f(x_n)$	probability density function of x_n
$f(x_1, x_2,\ldots,x_n)$	joint probability density function
G	independent anisotropic standard normal random field
H	ranked vectors based on the measured data **D**
L	lower triangular matrix decomposed from $\mathbf{M}_a$
$\mathbf{M}_a$	autocorrelation matrix of a random field
$\mathbf{M}_c$	cross-correlation matrix for multi-variate random fields
p_f	probability of failure
p_{20}	probability of plastic limit less than 20%
s	random variate from Chi-Square distribution with υ degrees of freedom
T	n correlated standard uniform random vectors following t-distribution
U	n correlated standard uniform random vectors with a target copula
W	marginal distributions of the measured data **D**
x_n	n^{th} random variable
Z	standard normal vector with independent elements
θ	copula parameter
φ	generator of the Archimedean copula
ρ_{ij}	cross-correlation coefficient between i^{th} and j^{th} random vectors
α, β	fitting parameters of Gamma distribution
κ	number of copula parameters
ρ_s	auto-correlation coefficient of a random variable at two locations
δ_1	major scale of fluctuation
δ_2	minor scale of fluctuation
υ	degree of freedom for t-distribution
ϕ	friction angle of soil
γ	unit weight of soil

A joint probability density function (PDF) or a joint cumulative distribution function (CDF) is needed to describe a problem involving several random soil parameters [35]. In most cases, only the marginal distributions for the separate parameters and the correlation coefficients among the parameters are known. Fig. 1 shows the joint PDF isolines of standard normal variables N1 and N2 with four dependence structures. A similar procedure for producing PDF isolines can be followed with reference to Tang et al. [36]. Fig. 1a is the case when the two random variables are independent. Fig. 1b shows the case in which the joint PDF is a multivariate standard normal distribution with a Gaussian-dependence structure. Figs. 1c and d involve identical standard normal marginal distributions and the same correlation coefficient of 0.5, but one has a tail-dependent structure (Fig. 1c) and the other has a central-dependent structure (Fig. 1d). Previously, cross-correlated random fields were mostly limited to bivariate types considering a correlation coefficient, or trivariate types considering a matrix of correlation coefficients. The generation of multivariate cross-correlated random fields following a rigorous joint distribution function with different dependence structures remains a difficult issue and has not yet been explored.

In this paper, a generic method is proposed to generate multivariate cross-correlated random fields explicitly including possible dependence structures of the variables. The specific objectives of

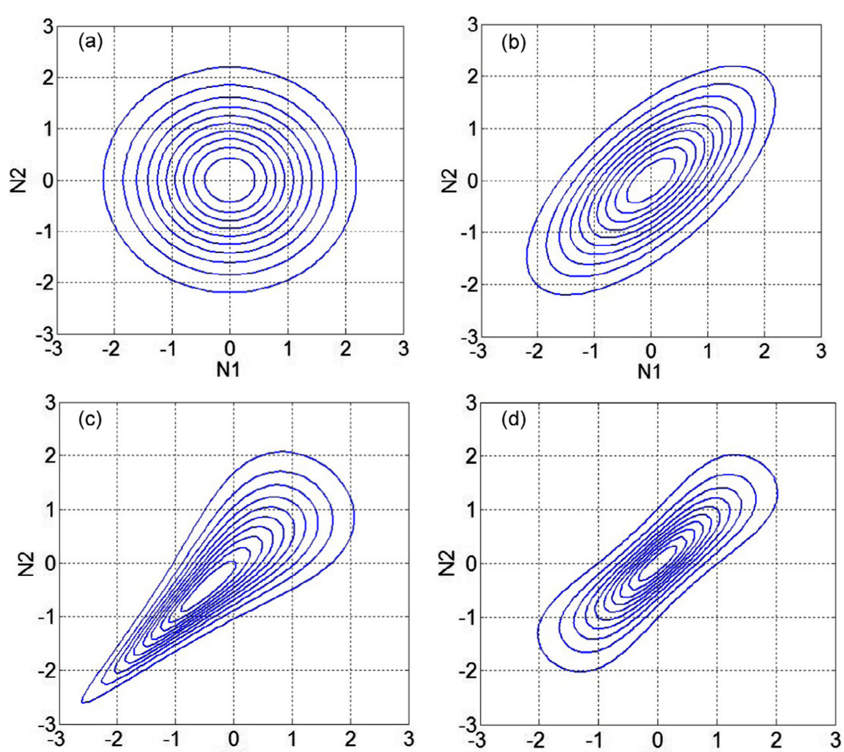

Fig. 1. Isolines of probability density functions for standard normal random variables: (a) independent; (b) Gaussian; (c) tail-dependent; (d) central-dependent.