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Research Paper

Lower bound limit analysis of wedge stability using block element method

Jianfeng Zhou, Junxing Wang *

State Key Laboratory of Water Resources and Hydropower Engineering Science, Wuhan University, Wuhan, Hubei 430072, PR China

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ABSTRACT

A lower bound limit analysis approach based on the block element method is proposed to analyze wedge stability problem. The search for the maximum value of the factor of safety is set up as a nonlinear programming problem. Sequential quadratic programming (SQP) algorithm from a reasonable initial value is applied to obtain the optimal solution. This approach provides a strict lower bound solution considering the sliding mode and rotation effect simultaneously. The deviations of the factor of safety between the present and traditional limit equilibrium methods are positively correlated with both the friction angle and the dip of the discontinuity surface.

1. Introduction

Wedge failure, caused by numerous intersecting discontinuities, can occur over a much wider range of geologic and geometric conditions than plane failures. It is an important component of slope failure and often occurs in both civil and mining engineering. Generally, the traditional method for wedge stability analysis, which is based on the limit force equilibrium and assumes that the mode of wedge failure is sliding and the direction of shear force is parallel to the line of intersection, has been studied by many researchers [19,15,16,14,37,28,13,18]. Significant progress in the process of the traditional limit equilibrium (TLE) has been made. However, the solution is not strict because it makes extra assumptions. The wedge failure mode is very complex in practice, which is not limited to the pure sliding failure only. Therefore, Wittke [38], Wittke [39], Chan and Einstein [2], Mauldon and Goodman [26], and Tonon [32] discussed the rotational stability of a rock block, but their studies were limited to special cases of rotation, and they did not provide a procedure that could handle general modes of simultaneous sliding and rotation. Yeung et al. [41] used three-discontinuous deformation analysis (3D-DDA) to work with this problem. However, the calculation of the safety factor, the inputs of complex parameters and the treatment of the contact problem are the main concerns. Therefore, to date, the 3D-DDA method has not been widely used for general wedge stability analyses in practice.

Obviously, the methods mentioned above all have drawbacks as discussed. To overcome the limitations of the existing limit equilibrium analysis methods and seek a method with more rigorous theory, Wang et al. [36] applied the classic upper bound (UB) theorem of limit analysis to obtain the UB solution of wedge stability. The limit analysis provides new insight for solving the wedge failure problem. Nonetheless, we must recognize that the UB method [36] still uses the translational sliding mode, and it did not consider the rotation effect. In addition, it should be stressed that by using lower or upper bound limit analysis together, the true solution can be bracketed into a small range. Generally, a lower bound solution is more valuable in practice as it results in a safe design. However, to the authors' knowledge, there is limited study on the implementation of the wedge stability problem using lower bound solutions to consider the sliding mode and the rotation effect simultaneously and strictly.

In general, the numerical methods commonly used for assessing the stability of slopes can be classified into the equivalent continuum approach and the discontinuous media approach. The former is usually implemented by the finite element method, whereas the latter mainly includes the discrete element method (DEM) [10], discontinuous deformation analysis (DDA) [29], block element

* Corresponding author.
E-mail addresses: zhoujf@whu.edu.cn (J. Zhou), jxwang@whu.edu.cn (J. Wang).

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method (BEM) [6] and rigid finite element method (RFEM) [20]. Based on plasticity limit theorems [12] and using numerical methods, the corresponding two main categories of limit analysis methods, namely, the limit analysis method based on finite elements [24,30,31,25,21,17,34] and the limit analysis method based on the discontinuous media approaches [42,3,40,23], have been studied by many researchers. Generally, the latter is more suitable for the application of rock slope because it can more easily handle rock slopes containing many discontinuities of various scale and orientation. As an important part of the discontinuous media mechanics system, the simple and strict mechanics concepts and the improved consideration of the special requirements from engineering practice such as the processing of complex structures with an irregular ground surface, various loading and construction processes lead to the popularity of the BEM. Engineering applications are given in [5,8,4,7]. For the reasons mentioned above, in this paper we attempt to combine the lower bound method and block element method to analyze the stability of a wedge slope.

The objectives of this paper are (a) to derive the formulation of a lower bound method for wedge stability analysis based on BEM, (b) to analyze the deviations of the factor of safety (Fs) and the reaction in the present and other methods, (c) to analyze the application of the present method to wedge stability calculation under the action of the anchor force and horizontal seismic force, and (d) to discuss the practical implications of the findings.

2. Numerical formulation

When establishing spatial force equilibrium equations and moment equilibrium equations for wedge stability problems, the forces acting on each discontinuity surface involve five unknowns: the normal component of reaction (R_N), the two shear components of reaction in different directions (R_{S1} and R_{S2}) and the acting point of the reaction vector, which involves two unknowns (x and y), as shown in Figs. 1 and 2. Subscripts L and R denote the left and right discontinuity surfaces, respectively. For a wedge with two discontinuities, there are ten unknowns, and the magnitude of Fs is an additional unknown in the wedge stability problem.

However, the number of force equilibrium equations and moment equilibrium equations for a wedge is only six. If assuming that the forces on both discontinuity surfaces follow the Mohr–Coulomb failure criterion, then two additional equations are available. Therefore, there is a total of eight equations with eleven unknowns. Generally, the TLE method [15] assumes that the mode of wedge failure is sliding and that the shear force direction is parallel to the line of intersection. Through these assumptions, six

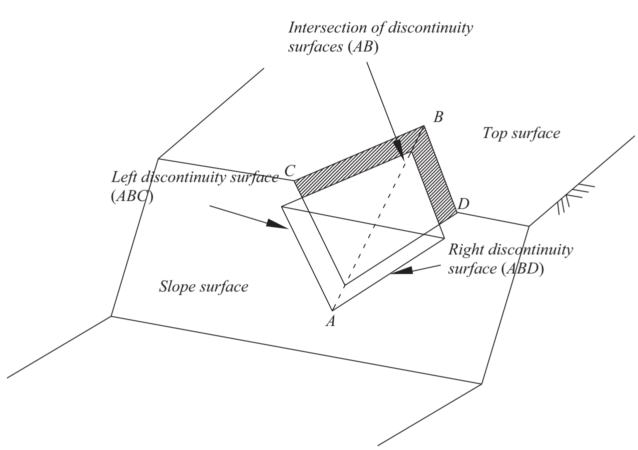

Fig. 1. Wedge slope of two slip surfaces.

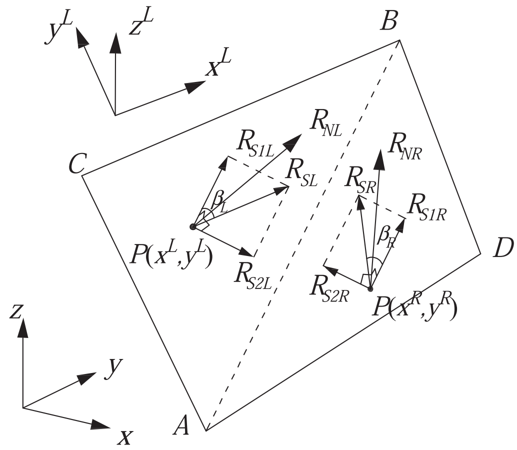

Fig. 2. Reactions on the slip surfaces.

unknowns and three moment equilibrium equations can be reduced. In other words, three unknowns will be reduced in total, and the question can be solved. Obviously, the wedge stability problem is statically indeterminate, which causes the limitations in existing limit equilibrium analysis methods.

The lower bound limit analysis provides new insight for solving the wedge failure problem. The lower bound theorem states that the collapse load obtained from any statically admissible stress field will underestimate the true collapse load. A statically admissible stress field is one that satisfies (a) the stress boundary conditions, (b) equilibrium, and (c) the yield conditions (the stresses must lie inside or on the yield surface in stress space) [30]. Based on lower bound limit analysis and BEM, the establishment of the present numerical method will be presented in the following.

2.1. Assumptions and coordinate system based on BEM

2.1.1. Assumptions

Based on the BEM, the following two assumptions are generally used in this paper: (1) The wedge is assumed to be rigid. (2) The interfaces are assumed to be isotropic, obeying the Mohr–Coulomb yield condition.

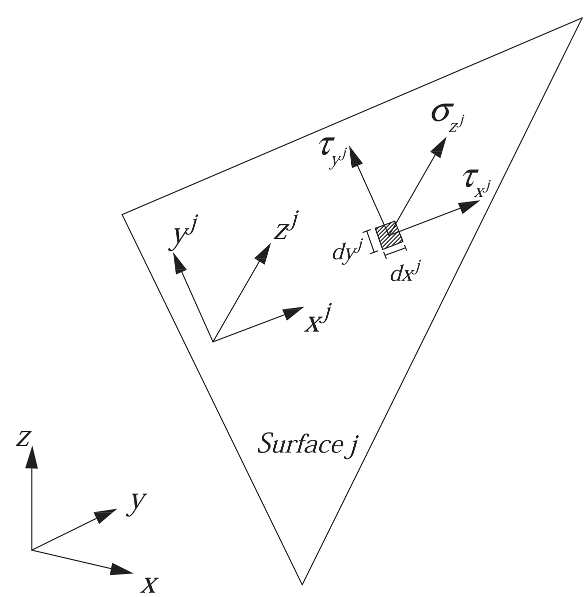

Fig. 3. Coordinates of structure surface *j*.