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Research Paper

Developing a 3D elastoplastic constitutive model for soils: A new approach based on characteristic stress

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ABSTRACT

This paper presents a new approach for the development of an elastoplastic constitutive model to predict the strength and deformation behaviour of soils under general stress conditions. The proposed approach was based on characteristic stress, which considers the effect of the intermediate principal stress on the material strength. Referring to the Cam-clay model, the shear dilatancy equation, plastic potential function and hardening parameter for the developed model were all derived using the characteristic stress. The model predictions indicated that the established model could quantitatively reproduce the negative dilatancy behaviour, positive dilatancy behaviour, and three-dimensional strength properties of soils.

1. Introduction

The Cam-clay model, which was established and developed by Roscoe et al. [1,2], is a classic model that includes five material parameters. This model can accurately predict the deformation behaviours and strength properties of normally consolidated saturated clay under traditional triaxial compression conditions. Moreover, the Cam-clay model is the first elastoplastic constitutive model for soils that can reflect the properties of negative dilatancy, volume yielding, and shear failure [3]. Since the development of the Cam-clay model, several other models [4–17] have been developed for soils, including sand and clay, based on the Cam-clay model.

In fact, soils exhibit different strengths under different stress paths [18]. The Drucker Prager strength criterion (DPC) was adopted for the shear failure of clay in the Cam-clay model. Experimental results have demonstrated that the DPC overestimates the shear strength of geomaterials, except under the traditional triaxial compression condition, and also results in an incorrect intermediate principal stress ratio under the plane strain condition [19]. In this regard, the calculated load capacity of soils tends to be inaccurate when the Cam-clay model is applied to engineering or practical applications. Similarly, models based on the Cam-clay model with no strength revisions can only be applied under the traditional triaxial compression condition.

The strength problem of the Cam-clay model originates from the fact that the model only has two stress-invariant parameters (p and q), and none of the other parameters or variables are used to reflect the three-dimensional (3D) strength of soils [20]. Here, $p = \sigma_{ij}\delta_{ij}/3$ is the mean stress, and $q = \sqrt{3/2}\sqrt{s_{ij}s_{ij}}$ is the general shear stress in the principal stress space, where σ_{ij} is the stress tensor, s_{ij} is the deviatoric stress tensor, and δ_{ij} is the Kronecker delta, $i,j = 1,2,3$.

Until now, there have only been two approaches to improving models using the DPC as the failure condition, such as the Cam-clay model, for use in general stress states: (1) modifying the shear dilatancy equation in the principal stress space and (2) combining other failure criteria with the established models. To reasonably describe the strength and deformation behaviours of soils under general stress conditions, different dilatancy equations were used to develop a yield function or plastic potential function [21]. Then, two shear dilatancy equations under traditional triaxial compression and traditional extension conditions were used to describe the dilatancy behaviour of soils [21]. A cumbersome process was used to switch the flow rules when the load was mutually changed from extension to compression or vice versa. Therefore, the constitutive model developed in this manner was rather complex. A dilatancy equation for both compression and extension conditions was proposed [22] using a micromechanics approach. The parameters in the dilatancy equation could not be easily determined even

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though the dilatancies of the soils were described uniformly. In addition, the established model still overestimated the strength under traditional triaxial extension, as illustrated by the comparisons between the test data and the predictions. A third stress-invariant parameter, i.e., Lode's angle θ, was introduced to the dilatancy equation to reflect the 3D strength properties of soils [20]. In this work, q was expressed as a function of p and θ, which is referred to as the $g(\theta)$ function [23]. This method has been widely used [24–31] to extend the constitutive models to the 3D stress space. However, the $g(\theta)$ function cannot reflect the stress-induced anisotropy of soils with the increase of the stress ratio q/p [32]. In addition, $g(\theta)$ is always highly complex, which makes the use of the established stress-strain relationships difficult.

On the other hand, a modified stress (t_{ij}) concept was proposed [33] based on the spatially mobilized plane (SMP) to describe the influence of the intermediate principal stress on the strength of the soils. Then, the t_{ij}-concept was combined with the Cam-clay model [34], and the Matsuoka-Nakai (MN) criterion was the critical state line of the Cam-clay model. However, the positive dilatancy will occur except under the traditional triaxial compression condition [35]. The original Cam-clay model cannot describe a positive dilatancy. Therefore, the t_{ij}-concept was not an efficient way to improve models for use in general stress states. Consequently, a transformed stress concept was proposed [36]. In this concept, the strength surface of a strength criterion was converted into a circular surface, which likes the DPC in the principal stress space. The transformed strength criterion can be applied to constitutive models using in general stress states. For instance, the MN strength criterion can better explain the high-quality test results of soils; therefore, the MN criterion was combined with the Cam-clay model based on the transformed stress concept. Then the transformed Cam-clay model can accurately describe the 3D strength properties and deformation behaviours of normally consolidated clay in complex stress states. Moreover, based on the transformed stress concept, the Lade strength criterion [37], the MN strength criterion and the generalized nonlinear strength theory [38] have also been used to describe the 3D strength properties of saturated or unsaturated soils [38–43]. The transformed stress concept is only a method to revise an existing constitutive model to make it suitable for general stress conditions; it is not a method to establish a 3D constitutive model. Moreover, the transformed stress must be derived for a certain strength criterion.

A characterized stress concept was proposed by Lu et al. [44] to uniformly explain the failure mechanism for the types of geomaterials. Using the octahedral plane under the action of $\widehat{\sigma}_i$ as the failure plane and the frictional rule to explain the failure of geomaterials, a nonlinear unified strength criterion (NUSC) was developed. The NUSC can be applied to describe the 3D strength property of different types of geomaterials. In this study, the concept of $\widehat{\sigma}_i$ will be further developed. Based on $\widehat{\sigma}_i$, a new modelling approach was proposed to establish a 3D elastoplastic constitutive model for soils by referring to the Cam-clay model. First, a new dilatancy equation was developed in the $\widehat{\sigma}_i$ space and was used to describe the dilatancy of soils with different intermediate principal stress effects. The dilatancy equation was then verified and further derived to obtain the plastic potential function. Then, a 3D elastoplastic constitutive model for soils was established based on the non-associated flow rule. Finally, the applicability of the established model along different stress paths was verified using experimental data obtained by the authors and data available from the literature.

2. Overview of the $\widehat{\sigma}_i$ and NUSC

In the theoretical approach to developing a strength theory, it has been hypothesized that the failure of geomaterials generally occurs in the failure plane [45]. The differences among the strength theories are the corresponding failure planes. On the other hand, unified strength theories could be applied to describe the intermediate principal stress effect for various kinds of materials. The existing unified strength theories were established by adjusting the position and outer normal direction of the failure planes, and the failure planes are all under the action of the principal stresses.

Another way to develop a unified strength theory was proposed by Lu et al. [44], and this method was based on the simplest failure plane (octahedral plane) and an adjust acting stress (called the characteristic stress). The DPC is the upper limit of all nonlinear strength theories in the deviatoric plane [42]; moreover, the failure plane of the DPC is the octahedral plane under the action of σ_i ($i = 1, 2, 3$). Hence, the upper limit of the characteristic stress is σ_i. The strength curve of the MN strength theory in the deviatoric plane circumscribes the six corners of the Mohr-Coulomb strength curve. It is the lower limit of all nonlinear strength theories in the deviatoric plane [42]. The σ_i-intercepts of the SMP are $k\sqrt{\sigma_1}$, $k\sqrt{\sigma_2}$ and $k\sqrt{\sigma_3}$ in the σ_i space. Therefore, the intercepts of the SMP on the $\sqrt{\sigma_i}$-axis are all equal to k. If the SMP under the action of $\sqrt{\sigma_i}(i = 1, 2, 3)$ is a plane, it will be an octahedral plane. The $\sqrt{\sigma_i}$ will be the lower limit of the characteristic stress. Unfortunately, the SMP expressed in the $\sqrt{\sigma_i}$ space is a curved surface.

Thus, the above idea was further developed. The characteristic stress is defined by combining the principal stress (σ_i, $i = 1, 2, 3$) with a parameter, β, to describe the intermediate principal stress effect:

$$\widehat{\sigma}_i = p_{\mathrm{a}}\left(\frac{\sigma_i}{p_{\mathrm{a}}}\right)^{\beta} \tag{1}$$

where p_{a} is the atmospheric pressure, which is used for the dimensionless transformation. The value of β is a constant for a certain material, and varies with different materials.

The NUSC was developed by assuming the octahedral plane as the failure plane and using the frictional rule as the failure mechanism for geomaterials. Therefore, the stress expressions of the DPC and the NUSC are similar, as shown in Table 1. In addition, for a certain material, the stress ratios, $\widehat{\eta}$, are equal for all of the failure states. Consequently, the failure stress ratios of two simple failure states are applied to determine the parameter β, and the failure states are the traditional triaxial compression condition and traditional triaxial extension condition. The two failure stress ratios are expressed as:

Table 1
Comparison between the DPC and the NUSC.

	DPC	NUSC
Strength expression	$q = Mp$	$\widehat{q} = M\widehat{p}$
Mean stress	$p = \sigma_{ij}\delta_{ij}/3$	$\widehat{p} = \widehat{\sigma}_{ij}\delta_{ij}/3$
General shear stress	$q = \sqrt{3/2(\sigma_{ij} - p\delta_{ij})(\sigma_{ij} - p\delta_{ij})}$	$\widehat{q} = \sqrt{3/2(\widehat{\sigma}_{ij} - \widehat{p}\delta_{ij})(\widehat{\sigma}_{ij} - \widehat{p}\delta_{ij})}$
Stress ratio	$\eta = q/p$	$\widehat{\eta} = \widehat{q}/\widehat{p}$
Failure stress ratio	$M = \frac{6\sin\varphi_c}{3-\sin\varphi_c}$	$M = 3\frac{(1+\sin\varphi_c)^{\beta}-(1-\sin\varphi_c)^{\beta}}{(1+\sin\varphi_c)^{\beta}+2(1-\sin\varphi_c)^{\beta}}$