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Research Paper

On the stress-force-fabric equation in triaxial compressions: Some insights into the triaxial strength

Xuzhen He [a], Guoqing Cai [a,*], Chenggang Zhao [a], Daichao Sheng [b]

[a] School of Civil Engineering, Beijing Jiaotong University, Beijing 100044, China
[b] ARC Centre of Excellence for Geotechnical Science and Engineering, The University of Newcastle, Newcastle 2308, Australia

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ABSTRACT

The strength of granular materials during triaxial compression is investigated via a grain scale analysis in this paper. A 3D Discrete Element Method (DEM) program provides the triaxial strength data and helps to validate the micromechanical analysis. Some standard methods in statistics are employed first to quantitatively examine the assumptions made when deriving the stress-force-fabric (SFF) equation. After careful validation, a more concise format for the SFF equation is proposed for triaxial compressions. With this SFF equation, the strength is found to be jointly contributed by the magnitudes of the contact force anisotropy and fabric anisotropy. The influence of the initial void ratio, confining pressure and loading direction on the development of contact force anisotropy and fabric anisotropy is examined and presented. With similar techniques, the "force" term in the SFF equation is further decoupled, and an equation is obtained such that it explicitly links the contact force term with the friction coefficient between grains, a tensor defined as a statistic of the normal contact forces and a tensor defined as a statistic of the mobilisation status of contacts. Based on this equation, another equation regarding the stress ratio of granular assembly is obtained, and it clearly indicates two sources that contribute to the phenomenological friction nature of granular assembly. These two sources are caused by the contact force at the grain scale. The first is the anisotropy of the average normal contact forces, and the second is the mobilisation of contacts.

1. Introduction

The modelling of granular materials, especially the prediction of their peak strength, is extremely important in various geotechnical problems. Granular materials are traditionally modelled as a continuum because the problem scale length is much larger than the grain scale length. At every continuum point, there are physical quantities that are actually the average of the grain scale information over a large number of grains and voids. The classic continuum mechanical descriptions, such as the yield surface, flow rule, and hardening rule, are summarized from phenomenological observations of shearing granular samples in the laboratory. These continuum models have been successfully used in a wide range of engineering problems [7,28,29,30].

With recent developments in imaging technologies [1] and grain-based numerical algorithms [3,14,24], direct observation and quantitative measurement of grain-scale information and processes offer researchers opportunities to study and inspect granular materials at the grain scale. The fabric, which is characteristic of the microstructure of granular materials at the grain scale, has been used as state variables for constitutive modelling [12,25,31]. In addition, micromechanical studies, which aim to model granular materials from a physics-based analysis at the grain scale, have attracted researchers' attention [5,9,27,32].

Rowe [22] was probably the first to study granular assembly with analysis at the grain scale. Micromechanical studies have become popular again with the observation of anisotropic strength for gravity-deposited sands [19], which is closely related to the initial anisotropy of fabric. Experiments [26] and numerical simulations [14] further reveal that shearing also causes the development of fabric anisotropy. Rothenburg and Bathurst [21] were among the first to approximate directional functions with the Fourier series. The Stress-Force-Fabric (SFF) relationship between the macroscopic stress tensor and explicit statistical parameters, such as the anisotropic parameters for the contact force and fabric, is introduced [20]. This type of analytical relationship provides valuable insight into macroscopic mechanical behaviour from the response of granular materials at the grain scale [2,4,23,11,15,17]. However, several assumptions made when deriving the SFF equation have not been assessed vigorously in early

* Corresponding author.
E-mail addresses: hexuzhen@outlook.com (X. He), guoqing.cai@bjtu.edu.cn (G. Cai), cgzhao@bjtu.edu.cn (C. Zhao), Daichao.Sheng@newcastle.edu.au (D. Sheng).

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studies. Some recent studies [13,16] have been conducted to re-evaluate these assumptions to provide more-solid ground for the equation and to facilitate further studies. In terms of applications, the strength of 2D direct shear or biaxial compression has been explored by incorporating SFF equations for ease of anisotropic description in 2D [3,14].

In this paper, some standard methods in statistics are employed first to quantitatively examine the assumptions made in the process to derive the SFF equation. After careful validation, a more concise format for the SFF equation is proposed in this paper for 3D triaxial compressions. With similar techniques, further analysis is then conducted on the "force" term in the SFF equation to explicitly include the friction coefficient between grains.

2. Discrete element modelling

DEM is a method to model the response of materials composed of massive discrete grains. With a system of discrete grains, contact laws between grains and specific boundary conditions, DEM has been successfully used in simulations of an extensive range of problems [3,8].

In this study, 3D spheres with different diameters are used to represent the granular system. However, two spheres with the same diameter are "clumped" together as a grain to consider the orientation of elongated grains. The distance between two clumped spheres is 1.5 times their common diameter. Additionally, the diameters of spheres are uniformly distributed between 0.03 mm and 0.07 mm.

A point contact assumption is made in DEM. In the contact law, the relative displacement along the normal direction $\Delta\vec{u}^n$ is calculated by the penetration depth, and an incremental algorithm is adopted in the calculation of the tangential relative displacement $\Delta\vec{u}^t$. The normal contact force is modelled with an elastic model $|\vec{f}^n| = K|\Delta\vec{u}^n|$. K is the normal stiffness and is calculated as $\frac{2E_1E_2r_1r_2}{E_1r_1+E_2r_2}$. Here, E and r are the Young's modulus and the radius of two contacting grains, respectively. A slip model is used for the tangential contact force $\frac{\vec{f}^t}{|\vec{f}^n|} = -\min\left(\xi\frac{|\Delta\vec{u}^t|}{|\Delta\vec{u}^n|}, \mu\right)\frac{\Delta\vec{u}^t}{|\Delta\vec{u}^t a|}$. Here, ξ is the ratio between tangential stiffness and normal stiffness, and $\mu = tan\phi$ is the friction coefficient at the contact. The slip model ensures that the ratio between magnitudes of tangential contact force and normal contact force cannot exceed the friction coefficient. In this study, because we are only interested in the relationship between the macroscopic state and the statistical state of grain scale information, a typical value of 150 MPa is chosen for E. The ratio ξ can determine the distribution of external loads between the normal contact force and the tangential contact force. A typical value of 0.5 is chosen in this study. The response of the granular system is largely influenced by the friction angle. For example, during isotropic compression, if a small friction angle is used, the resistance to compaction will be small, and a denser specimen can be obtained. Therefore, specimens with different initial void ratios can be prepared with this method. During triaxial compression, the friction angle between grains is chosen as 30°, and there is no friction between walls and grains in this study.

Triaxial compression is conducted on granular cubes with various initial states in DEM (Fig. 1). Isotropic compaction is first conducted until the isotropic confining pressure p'_0 is reached and the packing is stable. Then, the same constant stress is maintained on lateral walls (perpendicular to the X- and Y-axes), while the top and bottom walls (perpendicular to the Z-axis) load the packing at a fixed velocity of 0.06 m/s until the critical state. The boundary conditions are fulfilled by moving the walls in the explicit DEM algorithm. For walls where target stress is expected (the lateral walls), the overall stiffness between each wall and its contacting grains is calculated first, and thus, the moving distance and velocity of the wall can be estimated from the forces from grains, the external forces (the expected stress multiplied by the area) and the stiffness. Additionally, a maximum moving velocity and a damping technique are implemented to ensure the stable movement of the walls. In the damping technique, the actual moving distance of the wall at each step is estimated by the calculated moving distance at both the current step and the previous step.

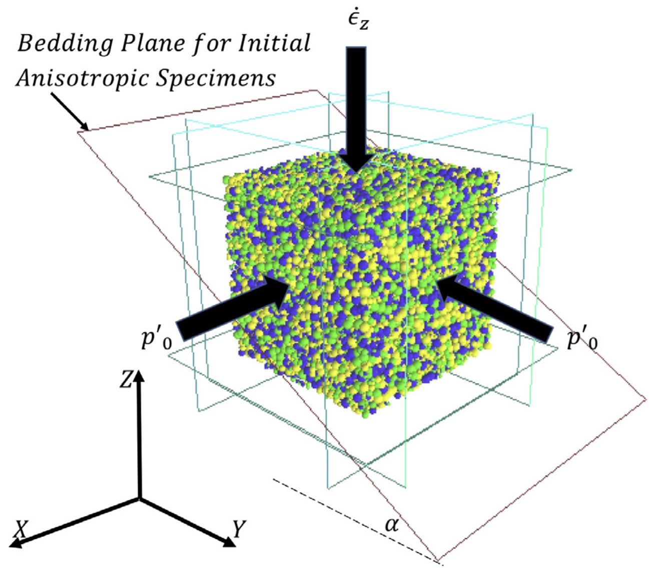

Fig. 1. An illustration of the triaxial compression in 3D DEM simulations.

The edge length of cubes is approximately 3 mm, and there are approximately ten thousand 'clumped' grains in each simulation. Specimens are prepared by inserting gains into a cube. No contacts between grains exist at first. The diameter and orientation of each grain are chosen randomly. Isotropic compaction is then conducted on these grains to obtain packed isotropic specimens, and specimens with different initial void ratios are obtained by changing the friction coefficient during compaction. Gravity deposition is simulated to have initial anisotropic specimens. Triaxial compression is conducted in different loading directions with respect to their bedding plane. The bedding plane is made parallel to the X-axis and inclines a loading angle α with the Y-axis (Fig. 1). The initial states of specimens are summarized in Table 1. There are three initial isotropic specimens, but each at a different density. All specimens are described by their relative void ratio at a 100 kPa confining pressure and the loading direction as in Table 1. As for the initial anisotropic specimens, all dense specimens are prepared with an initial void ratio of approximately 0.58, and loose specimens have a higher void ratio of approximately 0.74. All specimens are compressed under a p'_0 of 100 kPa, 200 kPa or 300 kPa. In this paper, the convention in soil mechanics that compressive stress and strain are positive is adopted. In the triaxial setting, the confining pressure is $p = (\sigma_1 + 2\sigma_3)/3$, and the deviatoric stress is $q = \sigma_1 - \sigma_3$.

3. Analysis of stress at the grain scale

The procedure to derive the Stress-Force-Fabric (SFF) equation is extensively discussed in the literature [17,18,21]). Some key assumptions and deriving processes are recapped in this section to introduce the notations used in this paper and, most importantly, to investigate the range of errors caused by these assump-