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Support performance of functionally graded concrete lining

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HIGHLIGHTS

- Functionally gradient is introduced into support of circular underground space.
- Elasto-plastic analysis and model tests on two-layered circular lining are conducted.
- Functionally graded lining has higher bearing capacities than single-layered lining.

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ABSTRACT

By inverse analysis, the concept of functionally gradient is introduced into thick-walled lining to improve its supporting performance. Analysis shows that an ideal functionally graded lining has a higher elastic limit than a conventional single-layered lining. The ideal functionally graded lining model requires that the Young's modulus of the material should monotonically increase with the increase radius, which cannot be achieved in a single-layered lining. A composite lining composing of multi layers that have different Young's modulus can approximately function as an ideal functionally graded lining. Such a two-layered lining was studied and was compared with a single-layered lining. The study involved elasto-plastic analysis and model testing. The elasto-plastic analysis shows that the elastic load bearing capacities of the ideal functionally graded lining and the two-layered functionally graded lining are higher than the traditional single-layered lining. The model test results confirmed the analytical conclusion.

1. Introduction

Concrete thick-walled lining is a rock support structure widely used in circular openings of underground excavation, like shafts, tunnels, etc. The in situ stresses in rock increase with depth. For instance, the in situ vertical stress is approximately 27 MPa in depth 1000 m [1]. In deep rock excavations, compressive failure often occur in conventional concrete lining, staring in the inner boundary and extending outward (see Fig. 1). In order to improve the bearing capacity of the lining and to insure the lining works within the limit of safety, two means are usually used: either to increase the lining thickness or to enhance the material strength. For the first means, the work stress in the lining can be reduced, but the reduction is limited. Furthermore, it is not economical. Statistics show that rock excavation comprises 40%–60% of the total cost in shaft construction. For a shaft extended to a depth of 1000 m, a decrease of 10 mm in lining thickness will lead to a reduction of the total cost by 1% for reinforced concrete and 0.25% for plain concrete. By the second means, that is, enhancing the strength of concrete, the high grade cement and the demand of strictly controlled construction technology would lead to high cost on one hand and on the other hand the increase in the strength may change the deformation behavior of the concrete and the material becomes more brittle. Both theoretical studies and empirical experience show that it is not cost effective to improve the performance of concrete lining either by increasing the lining thickness or by enhancing the material strength.

The concept of Functionally Graded, hereafter FG, is introduced in this paper to improve the performance of concrete lining. The concept was proposed in 1984 by Japanese materials scientists as a means of preparing thermal barrier materials. The idea of FGM is to adapt the loading condition and thermal conductivity by gradiently changing the composition, microstructure, porosity, etc. of the material [2]. Compositional micro/macrostructure gradient cannot completely dismiss undesirable effects such as stress con-

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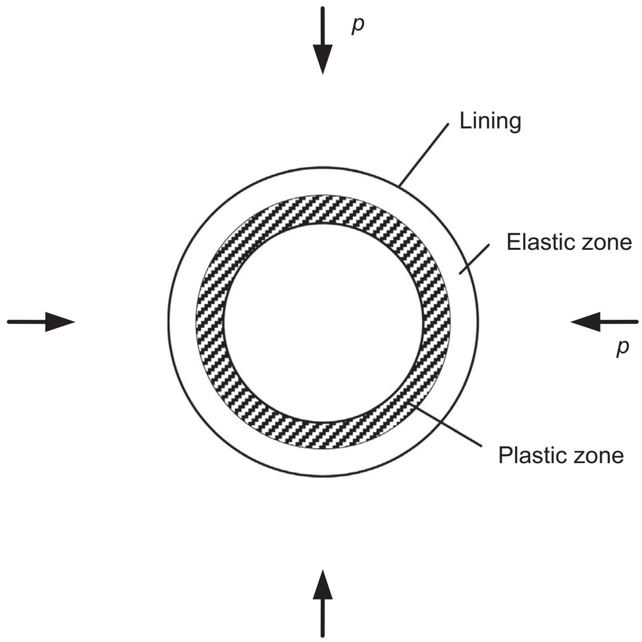

Fig. 1. Conventional single-layered lining under hydrostatic in situ stress p.

centration, but can improve the performance of the material [3]. Application of the concept has been extended to areas of manufacturing components of chemical plants, solar energy generators, heat exchangers, nuclear reactors and high efficiency combustion systems [4]. Efforts are also carried out on the functionally graded performances of construction materials [5,6]. Shen et al. [7] developed a functionally-graded material system with a spatially tailored fiber distribution to produce a four-layered, functionally-graded fiber-reinforced cement composite. Dias et al. [8] discussed the use of statistical mixture designs to choose formulations and presented ideas for the production of functionally graded fiber cement components. The study introduced in this paper aims to improve the stress distributions in composite concrete lining by changing the mechanical properties in the material layers. To achieve this, inverse analysis is first carried out to find the required variation pattern of the Young's modulus with respect to radius r. An ideal variation model for the Young's modulus is then established for functionally graded lining. A two-layer lining under hydrostatic pressure is used to demonstrate the stress distributions in functionally graded lining. Finally, the results of physical model tests were introduced to verify the theoretical results.

2. Inverse analysis of ideal functionally graded lining (FGL)

There have been many analytical results on FG thick-walled hollow cylinder subjected to temperature loads or stresses. In those works, material parameters are pre-assumed to be a function of the radius. Usually, the Poisson's ratio is set as a constant but the Young's modulus or shear modulus are defined as functions of the radius like linear variation [9,10], power law variation [11–21], exponential variation [14,17,18,22] and other forms [23,24]. All the above-mentioned works mainly calculate stresses and displacement distributions in a FG thick-walled hollow cylinder under given Young's modulus E(r) or shear modulus G(r). Obviously, the stresses and displacement distributions are dependent on E(r) or G (r). In the study, the desired distribution is pre-assumed with the Young's modulus E(r) being undetermined. The E(r) is determined by back-calculations according to the stresses distribution and loadings. This process is the so-called inverse analysis [25]. In our study, the FGL is subjected to a hydrostatic pressure p with a plane strain condition. Compressive stress is defined as positive in this paper.

2.1. Basic assumptions

If we want to take full advantage of the load-bearing capacity of the material, the ideal state is that the whole lining enters into plastic yielding at the same time. For the sake of simplicity, the lining concrete is assumed to obey the Tresca criterion in this work. The strength of the lining material is equal to $\sigma_1 - \sigma_3 = \sigma_c$, where σ_c is the Uniaxial Compressive Strength (UCS) of the material. Because of the axial symmetry of the problem, the tangential and radial stresses, σ_θ and σ_r, in the cross section plane of the opening represent the major and minor principal stresses, that is, $\sigma_1 = \sigma_\theta$ and $\sigma_3 = \sigma_r$. The FGL is so loaded that the difference $(\sigma_\theta - \sigma_r)$ is equal to a constant c. The lining is in elastic state when $c < \sigma_c$, and completely enters into plastic state when $c = \sigma_c$. To achieve a constant stress difference $(\sigma_\theta - \sigma_r)$ in the entire lining, the Young's modulus E has to vary with the radius r. The required variation for E will be derived in the subsequent sections in this paper.

2.2. Basic equations and solution

Assume that the Young's modulus E(r) is a function of radius r in order to achieve a uniform stress state $(\sigma_\theta - \sigma_r) = c$. For the plane-strain axisymmetric problem illustrated in Fig. 1, the strains, constitutive and equilibrium equations are expressed as

$$\varepsilon_r = \frac{du}{dr} \tag{1}$$

$$\varepsilon_\theta = \frac{u}{r} \tag{2}$$

$$\varepsilon_r = \frac{1-\mu^2}{E(r)}\left(\sigma_r - \frac{\mu}{1-\mu}\sigma_\theta\right) \tag{3}$$

$$\varepsilon_\theta = \frac{1-\mu^2}{E(r)}\left(\sigma_\theta - \frac{\mu}{1-\mu}\sigma_r\right) \tag{4}$$

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{5}$$

where ε_θ and ε_r are respectively the tangential and radial strains, u is the radial displacement and μ is the Poisson's ratio.

Substituting $\sigma_\theta - \sigma_r = c$ into Eq. (5) and solving the partial differential equation gives the expression of the radial stress as

$$\sigma_r = A + c\ln r \tag{6}$$

where A is obtained as $A = -c\ln R_{in}$ by substituting the stress boundary condition on the inner boundary into Eq. (6), that is, letting $r = R_{in}$ and $\sigma_r = 0$. Then Eq. (6) becomes

$$\sigma_r = c\ln(r/R_{in}) \tag{7}$$

The tangential stress is expressed, by considering $\sigma_\theta = \sigma_r + c$, as

$$\sigma_\theta = c[1 + \ln(r/R_{in})]. \tag{8}$$

As mentioned previously, when the constant c reaches the UCS of the lining concrete, the whole lining enters into plastic state at the same time. Linings that satisfy Eqs. (7) and (8) are called ideal FGLs (see Fig. 3(B)). For an ideal FGL, the elastic limit p^e_{IFGL} and the ultimate plastic pressure p^p_{IFGL} actually are equal, which can be obtained by considering the stress boundary condition on the outer boundary $r = R_{out}$ where $\sigma_r = p$,

$$p^e_{IFGL} = p^p_{IFGL} = \sigma_{cIFGL}\ln(R_{out}/R_{in}) \tag{9}$$