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Study of orthotropic pin fin performance through axisymmetric thermal non-dimensional finite element

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ABSTRACT

An axisymmetric thermal non-dimensional finite element is used to study the effects of different parameters on pin fin performance. It is observed that the fin performance may be improved by selecting an appropriate orthotropic thermal conductivity ratio. The thermal interface between fin and base plate may have considerable effects on overall heat dissipation, which is minimized by using appropriate thermal interface materials (TIM); nevertheless the minimized interface resistance depends on TIM thermal conductivity and its layer thickness. Effects of these two parameters on pin fin performance are studied in somewhat more detail. The thermal efficiency of fin may further be degraded by scale deposition on its surface, therefore the effect of scale deposition on a pin fin with TIM is also studied. All these above investigations are carried out using non-dimensional FE formulation, which directly provides the dimensionless results for a class of fin problems that become too complex for a dimensionless solution in a closed form.

1. Introduction

Application of pin fin as a heat sink is a well known problem therefore pin fin analyses may be found in the literature [1–4] as well as the derivation of temperature and heat flow equations for two-dimensional isotropic pin fins [4–6]. The impact of orthotropic thermal conductivity on the thermal performance of polymer composite fins has been studied by few researchers. Bhadur et al. [7] derived the closed-form solutions for temperature distribution and heat transfer from orthotropic pin fins; however, their results cannot be reduced to classical isotropic insulated-tip and convective-tip two-dimensional solutions. Zubair et al. [8] derived the generalized analytical solutions for temperature distribution, heat transfer rate, fin efficiency and fin effectiveness for orthotropic two-dimensional pin fins subject to convective-tip boundary condition. Their solution may be reduced to several special cases which also includes the insulated-tip boundary condition.

The pin fin may be installed by fusion or stud welding to the heat source, thus forming a continuous thermally conductive path for heat rejection. However welding is not possible on non-metallic heat source, therefore pin fin must be attached to the heat source by some other methods. The contact formed at fin-base plate interface consists of discrete micro contact spots resulting from surface roughness thus with trapped air in interstitial gaps. The interface resistance is therefore higher than heat sink element that can be overcome through thermal interface materials (TIM), which eliminate the air gaps by conforming to the rough and uneven mating surfaces, having a thermal conductivity higher than air. Theoretical as well as numerical approaches have been used to model the thermal contact resistance. A brief historical review of different contact resistance estimation is presented in reference [9]. The contact resistance models are also reviewed in Refs. [10–12], whereas theoretical models are presented in Refs. [11–17]. Numerical approaches have also been used; a thermo-mechanically coupled contact element is presented in Ref. [18]. A general three-dimensional thermal contact resistance finite element is presented in Ref. [19]. The finite element analysis has also been used to investigate the thermal contact resistance [20]. However all these approaches give an appreciable consideration to the resistance caused by imperfect contact between the two surfaces, which as discussed earlier, is overcome by using a TIM. TIM replaces the air contained in the gaps at non-conforming surfaces thus reduces the interface resistance due to its high thermal conductivity. The interface at fin–TIM and TIM–base plate may considered to be perfect, thus the governing parameter for the thermal interface resistance would be TIM thermal conductivity and layer thickness. This approach is adopted in this paper to model the thermal interface resistance offered by TIM.

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Nomenclature

h	Convective heat transfer coefficient
k	Thermal conductivity
q	Heat flux
r	Spatial variable in radial direction
s,t	Natural coordinates
t	Thickness
z	Spatial variable in axial direction
L	Fin length
R	Fin radius
T	Temperature

Non-dimensional variables

$\bar{q}^*$	Specified heat flux
$\bar{r}$	Spatial variable in radial direction
$\bar{z}$	Spatial variable in axial direction
Bi	Biot Number
θ	Temperature
η	Efficiency
ε	Effectiveness
ζ	Aspect ratio of the domain to be discretized.

Non-dimensional matrices and vectors

$\{\bar{\mathbf{f}}\}$	Element load vector
$[\bar{\mathbf{k}}]$	Element stiffness matrix
$[\bar{\mathbf{B}}]$	Flux-temperature matrix
$[\bar{\mathbf{D}}]$	Material property matrix
$[\bar{\mathbf{N}}]$	Shape function matrix
$[\bar{\mathbf{L}}]$	Geometric dimension ratio matrix
$\{\boldsymbol{\theta}\}$	Nodal temperature vector

Subscripts and superscripts

b	Base
f	Fin
h	Convection
i	Interface
q	Conduction
r	Radial
s	Scale
z	Axial
∞	Ambient
*	Specified value (for flux)
	Ratio (conductivity and thickness)

The objective of this paper is to study the effect of orthotropic thermal conductivity, interface resistance and scale deposition on the pin fin performance through non-dimensional axisymmetric finite element. The non-dimensional finite element formulation for axisymmetric thermal problems is first presented. The dimensionless finite element is capable of modeling conduction and convection phenomena for orthotropic materials.

2. Non-dimensional axisymmetric thermal finite element

In this section the non-dimensional finite element formulation for axisymmetric thermal element is derived through variational principle.

The governing equation of heat conduction in a cylindrical coordinate system for steady state is [21]:

$$\frac{1}{r}\left[k_r\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right]+k_z\frac{\partial^2 T}{\partial z^2}=0 \tag{1}$$

The boundary conditions are

$$\begin{aligned} &T=T_b \text{ on } S_1\\ &k_r\frac{\partial T}{\partial r}\hat{l}+k_z\frac{\partial T}{\partial z}\hat{n}+q^*=0 \text{ on } S_2\\ &k_r\frac{\partial T}{\partial r}\hat{l}+k_z\frac{\partial T}{\partial z}\hat{n}+h(T-T_\infty)=0 \text{ on } S_3 \end{aligned} \tag{2}$$

where, k_r and k_z are the thermal conductivities in r and z-directions, h is the coefficient of convective heat transfer, T_b and T_∞ are specified and ambient temperatures respectively, q^* is the specified heat flux and $\hat{l},\hat{n}$ are the surface normals. S_2 and S_3 are separate surface areas over which heat flux q^* (q^* is positive into the surface) and convection loss $h(T-T_\infty)$ are specified because they cannot occur simultaneously on the same surface.

The non-dimensional form of the governing equation and boundary conditions may be obtained by defining following non-dimensional parameters:

$$\theta=\frac{T}{T_\infty},\bar{r}=\frac{r}{L_r},\bar{z}=\frac{z}{L_z},\mathrm{Bi}_r=\frac{hL_r}{k_r} \text{ and } \mathrm{Bi}_z=\frac{hL_z}{k_z} \tag{3}$$

where θ is the non-dimensional temperature, $\bar{r}$, and $\bar{z}$ are non-dimensional spatial variables, L_r, and L_z are the maximum dimensions of the domain along r and z-directions respectively and Bi_r and Bi_z are Biot numbers.

Introducing the non-dimensional parameters in governing Eq. (1) and boundary conditions (2) leads to the following dimensionless forms:

$$\frac{1}{\bar{r}}\left[\frac{1}{\mathrm{Bi}_r}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\frac{\partial\theta}{\partial \bar{r}}\right)\right]+\frac{1}{\zeta}\frac{1}{\mathrm{Bi}_z}\frac{\partial^2\theta}{\partial \bar{z}^2}=0 \tag{4}$$

and,

$$\begin{aligned} &\theta=\theta_b \text{ on } S_1\\ &\frac{1}{\mathrm{Bi}_r}\frac{\partial\theta}{\partial\bar{r}}\hat{l}+\frac{1}{\mathrm{Bi}_z}\frac{\partial\theta}{\partial\bar{z}}\hat{n}+\bar{q}^*=0 \text{ on } S_2\\ &\frac{1}{\mathrm{Bi}_r}\frac{\partial\theta}{\partial\bar{r}}\hat{l}+\frac{1}{\mathrm{Bi}_z}\frac{\partial\theta}{\partial\bar{z}}\hat{n}+(\theta-1)=0 \text{ on } S_3 \end{aligned} \tag{5}$$

where $\zeta=L_z/L_r$ is the aspect ratio of the domain and $\bar{q}^*=\bar{q}^*/hT_\infty$ is the non-dimensional specified heat flux.

The variational principle may be used to obtain the element stiffness matrix and load vector. The variational principle specifies a scalar quantity (functional Π), defined by an integral form for a continuum problem. The solution of the continuum problem is a function that makes Π stationary with respect to arbitrary changes in it [22].

The functional for heat transfer problem, defined through (4) and (5), may be written as:

$$\begin{aligned}\overline{\Pi}=&\frac{1}{2}\iiint_{\overline{V}}\left[\frac{1}{\mathrm{Bi}_r}\left(\frac{\partial\theta}{\partial\bar{r}}\right)^2+\frac{1}{\zeta}\frac{1}{\mathrm{Bi}_z}\left(\frac{\partial\theta}{\partial\bar{z}}\right)^2\right]\mathrm{d}\overline{V}-\iint_{\bar{S}_2}\bar{q}^*\mathrm{d}\bar{S}\\&+\frac{1}{2}\iint_{\bar{S}_3}(\theta-1)^2\mathrm{d}\bar{S}\end{aligned} \tag{6}$$

where $\overline{V}$ and $\bar{S}$ are non-dimensional volume and surfaces given by:

$$\overline{V}=\frac{V}{L_r^2L_z} \text{ and } \bar{S}=\frac{S}{L_rL_z} \tag{7}$$

The minimization of $\overline{\Pi}$ with respect to θ yields: