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Modeling of turbine mass flow rate performances using the Taylor expansion

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ABSTRACT

The turbine is a key component of many equipment and systems, such as air cycle refrigeration and gas-turbine engines. Existing turbine mass flow rate models need to be improved to increase the prediction accuracy and extrapolation performance for control and diagnosis-oriented simulation. This work proposes a novel methodology for building a regression model, which makes use of the Taylor series to expand functions to deal with variables with small variation and develops a single partly empirical model to present a component performance map. With the methodology, a general regression model of mass flow rates of inward radial turbines is built. Measured data of a turbocharger turbine and a simple air cycle machine turbine are used for the regression analysis to validate the methodology and model. Model predictions agree with measured data very well, proving that the proposed methodology and the model are highly reliable. Comparison of the proposed model with the best existing model searched shows that the present model reduces the mean absolute percentage error by more than 50%, and has much better extrapolation performance as well.

1. Introduction

Air cycle refrigeration is a dominated refrigeration approach in aircraft environmental control systems (ECS), and the air cycle machine (ACM) is the main part of the air cycle refrigeration. In the ECS with the air cycle refrigeration, the compressed hot air expands through the ACM turbine, becoming cold air for cabin air conditioning and equipment cooling. When air expands through the ACM turbine, its heat energy is converted into the turbine shaft work. The turbine shaft work can be used to drive a compressor and/or fan to benefit the refrigeration cycle efficiency. If it is used to assist in the compression with a turbine-driven compressor, the turbine-compressor arrangement is called the bootstrap ACM. If it is used to drive a coaxial fan, the turbine-fan device is referred to as the simple ACM [1,2]. The turbine is also a key element of gas-turbine engines.

The mathematical model of turbine mass flow rate is the vital part of total models of equipment and systems using it. There are three dominant approaches for developing a mathematical model of turbine mass flow rates: Theoretical, empirical and partly empirical. The theoretical modeling is based on physical laws of mass, energy and momentum conservation. The high complexity of this modeling approach and its computational requirement makes it unsuitable for real-time dynamic simulation, especially control- and diagnosis-oriented simulation, where time efficiency is essential. Moreover, compact and computing efficient models are also preferred in design, analysis and performance simulation. Either empirical modeling or partly empirical modeling can yield simple and fast computing models. However, for the turbine mass flow rate the suitable model form of these kinds needs further investigation to increase prediction accuracies and extrapolation performances.

Plotting performance maps and generating look-up tables are the earliest approaches to process experimental data of turbines and compressors, which are now still in use [2,3]. Performance maps and look-up tables are helpful for design and performance analysis. For inclusion into system simulation models, however, maps are inapplicable and look-up tables are not well suited. Standard table interpolation routines are not continuously differentiable, extrapolation is unreliable and the table representation is not compact. Therefore, mathematical models from experimental data are desired.

One of the easiest methods to obtain empirical models from experimental data is curve-by-curve fitting, which is still used nowadays [3–5]. Tsujikawa [3] employed this method to generate mathematical models for the digital control of aircraft air cycle ECS. Kong et al. [4,5] assumed that turbine performance characteristics were the third-degree equations with the related function of the engine rotational speed versus the pressure ratio and the mass flow function. Turbine performance maps were generated by

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Nomenclature

a, a_i	constant ($i = 0, 1, 2, \ldots$)
A_{eff}	effective flow area, m^2
c_p	constant pressure specific heat, J kg^{-1} K^{-1}
D_1	blade wheel diameter, mm, m
G	turbine mass flow rate, kg s^{-1}
h	enthalpy, J kg^{-1}
k	isentropic exponent
n	blade rotational speed, r s^{-1}, RPM
n_t	rotational speed parameter
p	pressure, Pa
R	gas constant, J kg^{-1} K^{-1}
R^2	coefficient of determination
R_c^2	corrected coefficient of determination
s^2	residual mean square
T	temperature, K
u	blade peripheral velocity, m s^{-1}
v	air velocity, m s^{-1}
w	turbine shaft work, J kg^{-1}
x_0	turbine velocity ratio

Greek symbols

α	flow coefficient
α_1	blade fluid entry angle, deg
β_1	blade relative fluid entry angle, deg
γ	polytropic exponent
Δ	augment
π_t	turbine expansion ratio
ρ	density, kg m^3
ρ_t	degree of reaction
φ	nozzle velocity coefficient
ϕ	mass flow parameter, m s $\sqrt{K}$

Subscripts

0	turbine inlet
1	nozzle outlet, blade inlet
2	blade outlet
s	isentropic

integrating the turbine characteristic equations taken at each engine rotational speed.

Since the turbine mass flow rate is a function of the turbine expansion ratio and rotational speed parameter, a turbine mass flow rate map has a group of curves corresponding to each rotational speed parameter. Disadvantages of the curve-by-curve fitting are obvious. First, there are as many curve fit equations as curves on the map. Secondly, most of turbine operation conditions fall between curves, where there is no correspondent equation available. In this case, curve fit models of the two adjacent curves are used to calculate two adjacent values, thus needing interpolation which usually adds uncertainties. Besides, extrapolation of curve-by-curve fitting models is impossible when the mass flow rate is beyond the curve map range.

There are some attempts on using a single or a couple of partly empirical [6–10] or empirical [11,12] equations to represent a turbine flow performance map. Their primary purpose is to develop mean value models for control- or diagnosis-oriented simulations, where accuracies are compromised for tradeoff of computing time efficiency. The following section will review the two categories of models.

Moraal and Kolmanovsky [10] used a neural network with all inputs (speed parameter and pressure ratio) and one hidden layer with three neurons. They found that in order to get sensible extrapolation results for low turbine speeds, the neural network needed to be supplied with additional, artificial mapping points forcing the network to provide much lower turbine flows at these low speeds. Neural networks are becoming increasingly popular for curve fitting of compressor characteristic maps [13–16]. However, their usage in turbine performances is relatively rare.

The purpose of the present work is as the following:

(1) Reviewing existing empirical and partly empirical models representing a turbine flow performance map, excluding curve-by-curve ones.
(2) Proposing a novel methodology for building regression models of turbine characteristics in order to use a single equation to present a component performance map. The methodology develops a partly empirical regression model through rigorous analysis of related theoretical models and uses the Taylor series to expand functions to deal with variables with small variation (polytropic exponents for instance)
(3) Developing a general regression model of the mass flow rate of inward radial turbines using the methodology.
(4) To perform regression analysis of the proposed model using experimental data of a turbocharger turbine and a performance map of a simple ACM turbine and to compare the finalized regression model with existing ones to prove the reliability and correctness of the proposed methodology and regression model.

2. Review of empirical and partly empirical models of turbine mass flow rates

Extensive literature search for empirical and partly empirical models of turbine mass flow rates has been conducted. For empirical models, only polynomial expressions are found. This section summarizes searched results and compares existing models.

2.1. Partly empirical models

Partly empirical models are similar to the mass flow rate equation of steady flow through an adiabatic nozzle, with the effective flow area fitted using experimental data.

Canova et al. [6,7] use the following model to develop control-oriented submodels for the simulation of automotive diesel engines:

$$\phi = \begin{cases} A_{eff}\sqrt{\frac{2k}{R(k-1)}\left(\pi_t^{\frac{-2}{\gamma}} - \pi_t^{\frac{-r-1}{\gamma}}\right)} & \text{for} \quad \frac{1}{\pi_t} \geq \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{r-1}} \\ A_{eff}\sqrt{\frac{k(\gamma-1)}{R(k-1)}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{r-1}}} & \text{otherwise} \end{cases} \tag{1}$$

where ϕ is the mass flow parameter, π_t is the turbine expansion ratio, A_{eff} is the effective turbine area, R is the gas constant, k is the isentropic exponent, and γ is the polytropic exponent.

$$\phi = \frac{G\sqrt{T_0}}{p_0} \tag{2}$$

$$\pi_t = \frac{p_0}{p_2} \tag{3}$$