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Analytical solution of a circular opening in an axisymmetric elastic-brittle-plastic swelling rock

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ABSTRACT

Unconventional gas reservoirs such as coal and shale have been increasingly considered for methane production and CO_2 sequestration, over the last decades. In these reservoirs, methane and/or CO_2 are usually in an adsorbed state which is associated with swelling and/or shrinkage. There exist a number of experimental and theoretical studies on the effect of swelling and/or shrinkage in prediction of permeability, stress, and displacement distribution. However, most of these studies have only considered the elastic deformation of the reservoir. The plastic deformations within brittle reservoir rock can have significant implications for production, injectivity and stability and the wellbore and the reservoir. Therefore, development of improved models to estimate the distribution of stress and deformation around the wellbore is of great importance.

A large number of analytical solutions for axisymmetric opening problem have been presented in the literature where different models are used for material behaviour. This paper aims to include the effect of swelling/shrinkage in the elasto-plastic formulations around the wellbore. The reservoir is assumed to behave as a linear elastic material up to the yield point, which is identified by the Mohr-Coulomb failure criterion. The post-failure brittle behaviour of the rock is modelled by defining the residual strength parameters and employing a non-associated flow rule. Strains are decomposed into mechanical elastic, elastic swelling/shrinkage, and mechanical plastic parts. Although the swelling/shrinkage strains are considered to be elastic, their distributions are closely linked with distributions of plastic strains through sophisticated integral and differential relationships. The swelling/shrinkage is defined using a Langmuir-like curve, which is directly related to the pore pressure distribution within the reservoir. The model is then used to study the distributions of stress and strain around the wellbore, in both elastic and plastic zones and verified against a numerical solution. A parametric study is also conducted by defining different values for swelling parameters, and pre-failure and residual strength parameters. The provided model can be useful to estimate the failed zone around the wellbore, where the formation is irreversibly damaged. On the other hand, the estimated distributions and radial and tangential stress from this model can help develop new permeability models for unconventional reservoirs.

1. Introduction

Methane production from unconventional gas reservoirs such as coal and shale has significantly increased over the last decade, and they are also considered for CO_2 sequestration purposes. In these reservoirs, methane is an adsorbed state and its production is associated with a desorption-induced shrinkage. On the other hand, during CO_2 sequestration, adsorption of CO_2 in matrix of rock, leads to swelling of the matrix blocks. The multi-physical processes associated with CO_2 sequestration and methane production from unconventional reservoirs include thermo-chemo-hydro-mechanical interactions at different nano, micro, meso and macro scales. These processes and their interactions have been discussed and reviewed in by many investigations of coal (e.g. Busch and Gensterblum, 2011; Liu et al., 2011; Masoudian, 2016; Masoudian et al., 2013c) and shale (e.g. Huang et al., 2015; Meakin et al., 2013) and hydrate-bearing (e.g. Moridis et al., 2011; Rutqvist et al., 2012) reservoirs and they continue to attract many research efforts. Among all, the sorption-induced swelling/shrinkage has received a great attention, mainly due to its effect on permeability

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of the reservoir, and numerous models are suggested to relate the permeability of the unconventional reservoirs to the change in the stress distribution or the change in the fracture porosity as reviewed and discussed by many studies (e.g. Amann-Hildenbrand et al., 2012; Pan and Connell, 2012; Spencer, 1989). There exist a large number of experimental and theoretical studies on the effect of swelling and/or shrinkage in prediction of permeability and stress distribution within the reservoir (e.g. Chen et al., 2010; Connell, 2009; Cui et al., 2007; Masoudian et al., 2013a, 2016a; Sherwood and Bailey, 1994). However, most of these studies have only considered the elastic deformation of the reservoir. The plastic deformations within the reservoir rock can have significant implications for production, injectivity, and stability of the wellbore and the reservoir. The need for elastoplastic analysis and the significance of the plastic deformations around wellbores have been presented in the literature (e.g. Han and Dusseault, 2003). Recently, Lu and Connell (2016) have conducted a theoretical study and suggested that coal is more likely to undergo failure during gas production than other reservoirs. In addition, the stability of the wellbores is an important issue in any oil and gas production project. Therefore, improved models are needed to properly estimate the deformation and stress distributions around the wellbore.

A large number of analytical solutions for axisymmetric opening problem have been presented in the literature using different models of material behaviour, such as the elastic-perfectly plastic, elastic–brittle–plastic (Park and Kim, 2006) and elastic-strain softening/hardening (Chen et al., 2012; Zhang et al., 2012) models, along with the linear Mohr–Coulomb (M–C) and the nonlinear Hoek–Brown (H–B) (Sharan, 2003) and modified Cam Clay (Chen and Abousleiman, 2013) failure envelopes. Most of these solutions are developed for circular tunnels or boreholes in rocks with no fluid interactions and therefore, the stresses and deformations are simply mechanical. Three of the existing models are of more significance to this paper. Han and Dusseault (2003) provided a poro-elastic-perfectly plastic solution for a wellbore subjected to steady-state radial flow using M-C failure criterion. Park and Kim (2006) developed a closed-form solution for both M-C and H-B models to the problem of circular opening within an elastic-brittle-plastic continuum subjected to constant stresses at the boundaries. In addition, Zareifard and Fahimifar (2015) have also developed a poro-elastic-brittle-plastic solution for a deep tunnel in presence of steady-state groundwater flow. However, the existing models may not be directly applicable to unconventional gas recovery and/or CO_2 sequestration, mainly due to the complex nature of the physico-chemical phenomena in unconventional reservoirs. Firstly, swelling is a major player in hydro-mechanical performance of the unconventional reservoirs. Secondly, the experimental studies have revealed the brittle mechanism of failure in shale (e.g. Amann et al., 2011; Hull et al., 2015; Lisjak et al., 2014) and coal (Deisman et al., 2008; Gentzis et al., 2007; Masoudian-Saadabad et al., 2012; Masoudian et al., 2013b, 2014), and therefore, including the brittle failure mechanism in the models seems necessary.

In order to address the shortcomings of the existing models, this paper aims to include the effect of swelling/shrinkage strain and the brittle failure mechanism in poro-elastic-plastic solution of a wellbore. To achieve this, the reservoir was assumed to behave as linear elastic up to the yield point, identified by the linear Mohr–Coulomb (M–C) failure criterion. The post-failure brittle behaviour of the rock is considered through the use of residual strength parameters and employing a non-associated flow rule. In the plastic region, the total strains are decomposed into mechanical elastic, elastic swelling/shrinkage, and mechanical plastic parts. Although the swelling/shrinkage strains are considered to be elastic, their distributions can closely affect the distributions of plastic strains through sophisticated integral and differential coupled relationships. The swelling/shrinkage is defined using a Langmuir-like curve, which is directly related to the pore pressure distribution within the reservoir. The model was then used to study the distributions of stress and strain around the wellbore, in both elastic and plastic zones. The sensitivity of the elasto-plastic deformations was also studied by considering different values for swelling parameters, and pre-failure and residual strength parameters. The provided model can be useful to estimate the failed zone around the wellbore, where the formation is irreversibly damaged. On the other hand, the estimated distributions and radial and tangential stress from this model can contribute to developing new permeability models for unconventional reservoirs.

2. Problem definition

When considering the elastic-plastic deformations, it is of great importance to recognise the role and cause of different components of strain. The strain within the plastic zone has two components of elastic (ε^e) and plastic (ε^p) strains. However, the elastic strain can be decomposed into elastic mechanical strain, $\varepsilon^{e,m}$, and elastic swelling (or shrinkage) strain, $\varepsilon^{e,s}$, as stated below

$$\varepsilon = \varepsilon^e + \varepsilon^p = \varepsilon^{e,m} - \varepsilon^{e,s} + \varepsilon^p \tag{1}$$

Note that throughout this paper, the term ‘swelling strain’ is used to refer to $\varepsilon^{e,s}$, but it can represent the adsorption-induced swelling (e.g. CO_2 sequestration) or desorption-induced shrinkage (e.g. coalbed methane recovery), or their combination (CO_2-enhanced coalbed methane recovery). Also note that the adsorption induces a positive swelling strain while desorption induces a negative swelling (shrinkage) strain. This is the opposite of the common signs conventions in geomechanics and hence the sign for swelling strain term is negative in Equation (1).

On the other hand, the two most commonly-used failure criteria are the linear Mohr-Coulomb (M-C) and nonlinear Hoek-Brown (H-B), which can be written in light of Biot's effective stress definition ($\sigma'=\sigma-\alpha P$), as below

$$\begin{array}{lr} \sigma'_1 = \gamma\sigma'_3 + Y & M-C \\ \sigma'_1 = \sigma'_3 + \sqrt{m\sigma'_3\sigma_c + s\sigma_c^2} & H-B \end{array} \tag{2}$$

where σ'_1 and σ'_3 are the major and minor principal effective stresses at failure, respectively. σ_c is the uniaxial compressive strength of the intact rock, and m and s are the Hoek-Brown constants which depend on the properties of the rock. γ and Y can be defined based on the cohesion, c, and the friction angle, ϕ, of the rock as

$$\begin{array}{l} \gamma = \dfrac{1+\sin\phi}{1-\sin\phi} \\ Y = \dfrac{2c\cos\phi}{1-\sin\phi} \end{array} \tag{3}$$

Fig. 1 shows the plane view of a vertical borehole within a continuous homogenous isotropic elastic-plastic reservoir, under in-situ stress and pore pressure of σ_0 and p_0, respectively. The wellbore is subjected to a fixed internal pressure of p_w, and the radial stress is kept at its initial level at the outer boundary. The plastic zone is the region immediately in the vicinity of the wellbore in which the rock undergoes failure and plastic deformation, while the elastic zone is the region in which the stresses do not exceed the strength of the rock and therefore the deformations remain elastic.

Due to the axial symmetry of the problem, the major principal