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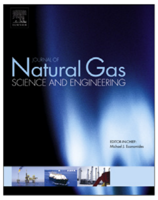

A new mathematical model to calculate sand-packed fracture conductivity

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ABSTRACT

The conductivity of sand-packed fractures is influenced by the fracture width and permeability. Proppants become rearranged, embedded in formations and even deformed under certain pressures. Thus, the fracture width, the porosity and the radius of seepage channels decrease with increasing pressure. Currently, sand-packed fracture conductivity is measured in the laboratory but rarely calculated by mathematical methods or based on ideal conditions, which may not always be true in practice. This paper demonstrates proppant rearrangement under low and high pressure and presents a rhombohedron model to calculate sand-packed fracture conductivity that considers the looseness coefficient, deformations of rock and proppants, particle size, crushing rate, number of proppant layers, closure pressure and so on. The mathematical model is based on the principle of elastic-plastic sphere contacts, the capillary bundle model and the Carman-Kozeny equation. In this case, the changing width and permeability of sand-packed fractures during compression can be calculated. Verification experiments and comparisons with existing models indicate that the mathematical models can reasonably predict the sand-packed fracture width and conductivity under real conditions.

1. Introduction

Hydraulic fracturing is the most widely used technology to stimulate oil and gas wells for more production or promote injection wells for more water injection by creating fractures with high flow conductivity. Many complex factors affect the conductivity of sand-packed fractures, while proppants are one of the main factors that we should consider because proppants become rearranged, embedded in formations and even deformed under increasing closure pressure, which decreases the fracture width and conductivity (Han et al., 2016). Research that examined sand-packed fracture conductivity has shown that suitable proppants with reasonable parameters (such as the particle size, material mechanical strength, sphericity, proppants' arrangement and so on; Pearson and Brannon, 2008) are required to maintain high conductivity in fractures. Therefore, studying the influencing factors of proppants on fracture conductivity is very important.

A few research projects studied sand-packed fracture permeability and can be classified into three types: experimental methods, the Carman-Kozeny equation or its derivative formulas, and other new mathematical methods. Experimental methods to measure fracture permeability are usually based on Darcy's linear percolation law or calculated by fitting empirical curves according to permeability experiments (Brace et al., 1968; Lin, 1977; Bourbie and Walls, 1982). Carman-Kozeny equation (Kozeny, 1927; Carman, 1937) or its derivative formulas (Gao and Li, 2013; Meng et al., 2014; Gao et al.,2014; Gao et al., 2015) are the most common mathematical methods to estimate sand-packed fracture permeability. These formulas contain the porosity of sand-packed fractures and the radius and tortuosity of seepage channels. A new mathematical method to calculate the permeability of sand-packed fractures was proposed by Berg (1970), which considered a particle size d_{50} and porosity, but applying the formula was difficult because the d_{50} and porosity parameters were obtained from crushed proppants (Zhang et al., 2015a,b). However, proppant crushing is a dynamic process during stress loading and is affected by many factors, such as the sanding concentration, arrangements and pressure.

Proppant embedment is an important component of the calculation of sand-packed fracture width and cannot be ignored. Huitt

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Nomenclature

P_c	Formation pressure, MPa
P_{c1}	Pressure on the top and bottom proppants, MPa
P_{c2}	Pressure on the middle proppants, MPa
w_{fo}	Original fracture width, mm
w_{f1}	Fracture width of the model, mm
w_f	Fracture width under a certain pressure, mm
Δw_f	Decrease in the rhombohedron model's fracture width under pressure, mm
n_1	Number of proppants in each layer
n_2	Number of proppant layers
N	Number of proppants in the model
Z_1	Vertical distance of M_1 and the rock plane
Z_2	Vertical distance of M_2 and the rock plane
δ_1/δ_2	Strain between two proppants on the middle proppants under P_{c2}/P_{c1}
δ_3	Strain between the proppant and the rock on the top and bottom proppants
V_{x1}	Volume of strain in the middle proppants
V_{x2}	Volume of strain in the top and bottom proppants
E_1/E_2	Young's modulus of the proppant/rock, GPa
S	Looseness coefficient
S_0	Empirical looseness coefficient
r_x	Radius of the seepage channel, mm
τ	Tortuosity of the seepage channel
R	Radius of the proppant, mm
ϕ	Porosity of the model, %
k	Permeability of the fracture, um^2
F_{RCD}	Conductivity of the fracture, $um^2 \cdot$cm
W_c	Strain under a certain pressure
W_1	Strain on M_1 in the proppant
W_2	Strain on M_2 in the rock
a_1/a_2	Radius of the contact surface
W_{10}/W_{20}	Strain on O in proppant 1/2
u_1/u_2	Poisson's ratio of the proppant/rock
η	Crushing rate of the proppants, %
V_1	Volume of the model
V_0	Volume of bondless packing proppants
C	Sphericity of the proppant

and Mcglothlin (1958), Lacy et al. (1998) and Lu et al. (2008) performed experimental research on proppant embedment. Their experimental results showed that the most important parameters were the closure pressure, sanding concentration and rock type. Furthermore, high pressure, low sanding concentration, or soft formations would lead to greater embedment, so applying high sanding concentration in soft formations or deep wells is recommended. In addition, some semi-empirical or empirical formulas were developed through abundant experiments. Huitt and Mcglothlin (1958) deduced a semi-empirical equation to calculate proppant embedment by considering the proppant concentration and overburden pressure. Volk et al. (1981) summarized an empirical formula according to experimental results, and the formula contained several factors, such as the proppant size, sanding concentration, proppant distribution, rock type, and so on. Guo et al. (2008) derived an empirical model to calculate proppant embedment by fitting their experimental data.

However, these semi-empirical and empirical equations have been derived from some specific conditions and are limited in general applications, so some mathematical methods have been created to calculate proppant embedment. One mathematic model was developed by Li et al. (2011) to calculate the proppant embedment with the particle size, closure pressure and Young's modulus of the rock. However, this method failed to consider proppant deformation and was only suitable under low pressure. Another mathematic model was provided by Guo and Liu (2012), which included elastic deformation and creep deformation. This model could couple fracture propagation and production prediction but first needed to create the function of closure pressure over time and ignored the effects of the proppants.

Currently, most of the methods to obtain sand-packed fracture conductivity are experimental methods (Cooke, 1973; Cutler et al., 1985; Schubarth et al., 1997; Lacy et al., 1997; McDaniel et al., 2010; Guo et al., 2011; Zhang et al.,2014a; Zhang et al., 2015a) and semi-empirical or empirical models. At the same time, some integrated mathematical models have been proposed to calculate the fracture conductivity, while most of these models were performed under ideal conditions or were not comprehensive enough to be appropriate under actual conditions. Therefore, mathematical models that considered the proppant size and rock type (Gao et al., 2014) and the development pattern when considering the crushing rate of proppants (Gao et al., 2015) were proposed based on the vertical displacement formula and Hertz's theory of elastic contact, respectively. However, both models lacked processes to calculate the radius of the capillary bundle, and the authors assumed that the stresses and strains on each proppant were the same, which is not true in reality. In addition, Meng et al. (2014) presented a model to calculate the fracture conductivity under ideal conditions, which ignored many important factors, such as the closure pressure and the deformation and embedment of proppants, so this model was not suitable to be applied in computations under real conditions.

Gao and Li (2013) derived a reasonable mathematical model to compute the conductivity based on Hertz's elastic contract theory and the Carman-Kozeny equation. The mathematical model showed details of computation for embedment and deformations of single-pattern and multi-pattern proppants, which were in a cubic arrangement, while the authors considered that the deformations for multi-pattern proppants were the superposition of each layer's deformation because of the strains of single-pattern proppants. Actually, the arrangements were not only cubic but also rhombic, and the stress on single-pattern proppants was also different than that of multi-pattern proppants. In addition, the equations of the permeability model were derived from the equations to compute the parameters when the closure pressure was zero and were closer to semi-empirical equations.

Zhang et al. (2015b) presented a new correlation to calculate shale fracture conductivity based on the population balance equation for particle size reduction and the Berg model to predict sand-packed permeability. This method was different from classical methods, which were based on Hertz's elastic contract theory and the Carman-Kozeny equation. This method also considered the rearrangement of proppants, proppant embedment and proppant crushing. The rearrangements included inter-granular frictional slippage and pore collapse, while the proppant strains were neglected when calculating the fracture width. The permeability was calculated by a modified formula that was derived from Berg's model but was still inconvenient to be applied in computations because of the aforementioned defects in Berg's model.

Yan et al. (2016) proposed a method to calculate sand-packed fracture conductivity through extrusion experiments of proppant pillars. This method used a new approach compared to the above methods, but its experimental values were not accurate because of