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# Algebraic solution of capillary tube flows Part I: Adiabatic capillary tubes

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ABSTRACT

Capillary tube flows have been solved through both numerical and analytical approaches. The former requires a reasonable understanding of the governing equations of heat and fluid flow, thermodynamic relations, numerical methods, and computer programming, and therefore are not the suitable approach for most refrigeration and air-conditioning practitioners. Some simpler procedures based on different strategies for analytically solving the capillary tube flow have been proposed in the literature, although iterative loops for calculating the mass flow rate are still required. The aim of this work is to advance a semi-empirical algebraic model to solve adiabatic capillary tube flows using a relatively simple set of thermodynamic relations and being explicit for the mass flow rate calculation. Comparisons with a comprehensive experimental data set, taken with the refrigerants HFC-134a and HC-600a, has shown that the model predicts more than 90% and nearly 100% of all data within ±10% and ±15% error bands, respectively.



## 1. Introduction

A capillary tube is a small bore pipeline connecting the condenser to the evaporator in small-scale refrigeration systems (i.e., cooling capacities below 5 kW). Liquid refrigerant flows into one end, and expands down to the evaporator pressure. In doing so it meters refrigerant at the desired mass flow rate. A capillary tube appears to be quite simple, but the refrigerant flow inside this component is rather complex. The flow offers several challenges for a phenomenological description: turbulence, phase-change, compressibility and non-equilibrium effects all occur in capillary tube flows. Due to the importance of capillary tubes to the refrigeration industry, models for sizing these components have been extensively proposed in the literature, spanning from empirical correlations [1–6] to first-principles simulation codes [7–12].

In general, the refrigerant flows in *adiabatic* capillary tubes have been modeled based on the following key assumptions: (i) the capillary is a straight, horizontal and constant cross-sectional area tube; (ii) the viscous compressible flow is one-dimensional in the axial direction; (iii) the pressure drop at the capillary tube entrance and exit sections is disregarded; (iv) the two-phase flow is homogeneous, and (v) the metastable flow is neglected. Hence, the governing equations, derived from the mass, momentum and energy conservation principles, can be expressed by the following set of ordinary differential equations [13]:

$$G^2 \mathrm{d}v + \mathrm{d}p + fG^2 v \mathrm{d}z/2D = 0 \tag{1}$$

$$\mathrm{d}h + G^2 v \mathrm{d}v = 0 \tag{2}$$

where the specific volume derivative, $\mathrm{d}v$, is obtained from

$$\mathrm{d}v = (\partial v/\partial h)_p \mathrm{d}h + (\partial v/\partial p)_h \mathrm{d}p \tag{3}$$

For a given mass flux $G$, there are three equations and four unknowns ($p$, $h$, $v$, $z$) and, therefore, one unknown must be chosen as the integration domain. Taking pressure as the integration domain, Eqs. (1)–(3) may be re-arranged as follows [13]:

$$\frac{\mathrm{d}z}{\mathrm{d}p} = -\frac{2D}{fG^2 v}\frac{1 + G^2[v(\partial v/\partial h)_p + (\partial v/\partial p)_h]}{1 + G^2 v(\partial v/\partial h)_p} \tag{4}$$

$$\frac{\mathrm{d}h}{\mathrm{d}p} = -\frac{G^2 v(\partial v/\partial p)_h}{1 + G^2 v(\partial v/\partial h)_p} \tag{5}$$

Eqs. (4) and (5) express the tube length and enthalpy variation with respect to the refrigerant pressure for any flow regime, respectively. The boundary conditions are the thermodynamic states at the entrance of the capillary tube (condensing pressure and subcooling) and the pressure at the capillary tube exit (evaporating or sonic pressure). It should be noted that there are three

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**Nomenclature**

| | |
|---|---|
| $D$ | capillary tube inner diameter (m) |
| $f$ | friction factor (dimensionless) |
| $G$ | mass flux (kg/s m$^2$) |
| $h$ | enthalpy (J/kg) |
| $L$ | capillary tube length (m) |
| $p$ | pressure (Pa) |
| Re | Reynolds number (=$4w/\pi D\eta_f$) (dimensionless) |
| RMS | root mean square error |
| $T$ | temperature (K) |
| $v$ | specific volume (m$^3$/kg) |
| $w$ | mass flow rate (kg/s) |
| $z$ | axial coordinate (m) |

*Greek symbols*

| | |
|---|---|
| $\eta$ | viscosity (Pa s) |
| $\Delta T_{sub}$ | subcooling degree at capillary inlet (K) |
| $\Phi$ | capillary tube constant (≈6.0) |

*Subscripts*

| | |
|---|---|
| $c$ | capillary tube, condensing |
| $e$ | evaporating |
| $f$ | flash point |
| $h$ | isenthalpic process |
| $i$ | inlet |
| $l$ | saturated liquid |
| $o$ | outlet |
| $p$ | isobaric process |
| $sat$ | saturation |
| $sonic$ | choked flow |
| $v$ | saturated vapor |

boundary conditions and only two equations, but one boundary condition (i.e., the exit pressure) has to be used for the mass flow rate calculation.

The solution algorithm requires the numerical integration of Eqs. (4) and (5) using a guessed mass flux, $G$, which is iteratively corrected since the flow might be choked at the capillary tube exit [13]. In addition, the thermodynamic properties, particularly the specific volume and its derivatives $(\partial v/\partial p)_h$ and $(\partial v/\partial h)_p$, and the friction factor ought to be calculated at every point of the solution domain. Because of these, the numerical approach is time consuming and requires some programming abilities. As an alternative, algebraic analytical solutions for capillary tube flows have been proposed in the literature, although most of them relied on iterative loops for calculating the refrigerant mass flow rate.

## 2. Existing algebraic solutions

Yilmaz and Unal [14] proposed an algebraic model for predicting the mass flow rate of pure refrigerant flows through adiabatic capillary tubes. The flow was regarded as isenthalpic, thus allowing Eqs. (1)–(3) to be re-written as

$$\frac{dz}{dp} = -\frac{2D}{fG^2 v}\left[1 + G^2(\partial v/\partial p)_h\right] \tag{6}$$

Assuming the liquid specific volume as constant and integrating Eq. (6) from the capillary tube entrance until the flash point (Fig. 1a), the liquid region length is then calculated from

$$L_{sp} = -\int_i^f \frac{2D}{fG^2 v}dp = \frac{2D}{f_{sp}G^2}\frac{p_i - p_f}{v_f} \tag{7}$$

In addition, Yilmaz and Unal [14] demonstrated that the two-phase specific volume along an isenthalpic line can be calculated by $v = a + b/p$, where $a = v_f(1-k)$, $b = v_f p_f k$, $k = 2.62\times10^5 p_f^{-0.75}$, $p_f = p_{vap}(h_{liq} = h_i)$, $v_f = v_{liq}(p_f)$, and thus $(\partial v/\partial p)_h = -b/p^2$. Therefore, the two-phase length was calculated integrating Eq. (6) from the flash point to the capillary tube exit,

$$\begin{aligned} L_{tp} &= -\int_f^e \frac{2D}{fG^2}\left(\frac{1 - G^2 b/p^2}{a + b/p}\right)dp \\ &= \frac{2D}{f_{tp}G^2}\left[\frac{p_f - p_e}{a} + \frac{b}{a^2}\ln\left(\frac{ap_e + b}{ap_f + b}\right) - G^2\ln\left(\frac{v_e}{v_f}\right)\right] \end{aligned} \tag{8}$$

Noting that the total tube length is given by $L = L_{sp} + L_{tp}$, Eqs. (7) and (8) can be re-written for the mass flow rate, $w$,

$$w = \frac{\pi D^2}{4}\sqrt{\frac{\frac{f_{tp}}{f_{sp}}\frac{p_i - p_f}{v_f} + \frac{p_f - p_e}{a} + \frac{b}{a^2}\ln\left(\frac{ap_e + b}{ap_f + b}\right)}{\frac{f_{tp}L}{2D} + \ln\left(\frac{v_e}{v_f}\right)}} \tag{9}$$

where $w$ is given in (kg/s). In the work of Yilmaz and Unal [14], both single and two-phase friction factors were calculated using the correlation proposed by Churchill [15], considering the following average viscosity for the two-phase region,

$$\eta_{tp} = \frac{8}{7}\eta_f\left[\frac{1 - (p_e/p_f)^{7/8}}{1 - p_e/p_f}\right] \tag{10}$$

where index $f$ refers to the flash-point properties. Such a position is determined either as shown in Fig. 1a, for subcooled, or Fig. 1b for two-phase flow inlet conditions.

The predictions of the formulation introduced by Yilmaz and Unal [14] have been compared with hundreds of in-house experimental data points taken with adiabatic capillary tubes and with the refrigerants HFC-134a and HC-600a. Table 1 summarized the operational and geometric conditions used during the experiments. As shown in Fig. 2, the Yilmaz–Unal model predicts 86.5% of all data within ±10% error bands.

In their work, Yilmaz and Unal [14] assumed the exit pressure as equal to the evaporating pressure, thus neglecting the occurrence of choked flow at the capillary outlet. Zhang and Ding [16] improved the Yilmaz and Unal's [14] model by considering the exit pressure to be $p_e = \max(p_{evap}, p_{sonic})$, where the sonic pressure at the capillary exit was obtained setting $dz/dp \to 0$ in Eq. (6),

$$p_{sonic} = G\sqrt{v_f p_f k} \tag{11}$$

Zhang and Ding [16] added several other contributions to Yilmaz and Unal's [14] formulation. Firstly, they refitted the correlation for $k(p_f)$, proposing $k = 1.63\times10^5 p_f^{-0.72}$. Also noting that Yilmaz and Unal's [14] formulation was implicit due to the friction factor dependence on the mass flow rate, they proposed a two-step predictor–corrector solution for Eq. (9) based on an approximated mass flow rate to calculate the friction factor.

Fig. 3 compares the predictions of the formulation introduced by Zhang and Ding [16] with the entire database, showing that the model predicts 82.9% of all data within ±10% error bands.

Later, Yang and Wang [17] derived an empirical $\pi$-type dimensionless correlation based on the formulation of Zhang and Ding [16], as follows,