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Estimation of the transient surface temperature and heat flux of a steel slab using an inverse method

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Abstract

In the steel industry it is of great importance to be able to control the surface temperature and heating- or cooling rates during heat treatment processes. An experiment was performed in which a steel slab was heated up to 1250 °C in a fuel fired test furnace. The transient surface temperature and heat flux of a steel slab is calculated using a model for inverse heat conduction. That is, the time dependent local surface temperature and heat flux of a slab is calculated on the basis of temperature measurements in selected points of its interior by using a model of inverse heat conduction. Time- and temperature histories were measured at three points inside a steel slab. Measured temperature histories at the two lower locations of the slab were used as input to calculate the temperature at the position of the third location. A comparison of the experimentally measured and the calculated temperature histories was made to verify the model. The results showed very good agreement and suggest that this model can be applied to similar applications in the Steel industry or in other areas where the target of investigation for some reason is inaccessible to direct measurements.

Keywords: Inverse heat conduction; Heat treatment; Heat transfer; Steel processing; Slab

1. Introduction

The motivations for using an inverse method may be manifold. Sometimes, especially in the field of thermal physics, one wants to calculate the transient temperature or heat flux on the surface of a body. This body may be a slab, or billet in metallurgical applications. However, it may be the case that the surface for some reason is inaccessible to exterior measurements with the aid of some measurement device. Such a device could be a thermocouple if contact with the surface in question is possible or a pyrometer if a non-invasive method is preferred. Sometimes though, these kinds of devices may be an inappropriate choice. It could be the case that the installation of any such device may disturb the experiment in some way or that the environment is chemically destructive or just that such instruments might give incorrect results. In the Steel industry, one large problem is oxide scale formed on the billets or slabs during reheating. The formation of oxide scale causes severe mass losses and affects product quality and economy. One of the main parameters that govern this growth is the surface temperature of the steel surface. However, the surface temperature is difficult to calculate using conventional calculation methods since many parameters inside the furnace chamber must be known for a complete model. The heat transfer mechanism inside the furnace to heat up the slab includes radiation from refractory which often includes a complex geometry and the convective contribution from burners which often includes a complex flow field. A heat transfer coefficient must thus be known. In

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Nomenclature

c	specific heat capacity (J/kg K)
f, f_{m}	exact and measured temperature histories (°C)
h, h_{m}	exact and measured heat flux histories (W/m^2)
L_i	distance between thermocouples or thermocouples/surface (m)
t	time (s)
T	temperature (°C)
$T0$	duration in time of experiment (s)

Greek symbols

α	thermal diffusivity (m^2/s)
λ	thermal conductivity (W/m K)
ρ	density (kg/m^3)

Subscript

i	integer

most existing models, the growth of oxide scale formation is omitted which makes the heat transferred to the slab erroneous. These kinds of studies are becoming increasingly more important as new combustion technologies are developed with requirements of being environmentally friendly and effective energy wise. In such situations one is directed to using an inverse method based on interior measurements in the body, and in which the desired temperature is calculated by a numerical procedure. The only parameters needed are the temperature histories at some selected locations inside the steel slab and the material properties of the slab.

In the literature [1,2], this family of problems is most often referred to as inverse heat conduction problems (IHCP). This paper is focused on the application of an inverse method developed in [3].

The aim of this paper is to demonstrate that, in practice, it is possible to determine the transient surface temperature, and heat flux, using a numerical method for solving the IHCP. That is, we present an experiment where the time dependent surface temperature of a slab is calculated on the basis of temperature measurements collected at selected points in the interior of the body. In our experiment the temperature data is recorded by thermocouples inserted into the material at appropriate locations beneath the surface we are interested in studying.

A common family of methods for solving the inverse heat conduction problems transforms the problem into an integral equation of first kind [4,5]. The drawback of these methods is that often the kernel in the corresponding integral equation is not known explicitly. This is the case, for instance, if the properties of the material, e.g. thermal conductivity and density, are dependent on the temperature, i.e. the problem is non-linear. In metallurgical applications, such as the experiments described in this paper, such methods cannot easily be used since the material properties of steel change considerably in the large temperature range present in our experiment.

The methods developed in [3] allow for problems in which the material properties depend on the temperature, i.e. the heat equation is:

$$\frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right) = \rho(T)\cdot c(T)\cdot\frac{\partial T}{\partial t}. \tag{1}$$

The dependence of the material properties λ, c and ρ, on T makes the problem non-linear. This approach is most suitable for the application described in this paper. A method for solving the IHCP using wavelets were proposed in [6], as was a method based on using the Fourier Transform. Both of these methods are based on the idea of rewriting the original partial differential equation as a system of ordinary differential equations. The ill-posedness of the problem is dealt with by approximating the time derivative by a bounded approximation. For the purposes of analyzing the data presented in this paper the method based on the Fourier Transform was selected. The Fourier method is simpler than the Wavelet based method, and both methods can be expected to provide similar results. An application to aluminum was done [7] in which a cooling problem was considered. This paper sets out to use the method for a heating problem in a temperature range relevant to reheating furnaces in the Steel industry [8].

This text is structured in the following way. A mathematical formulation of the IHCP is given in Section 2. In Section 3 we note that the problem is ill-posed and discuss briefly how the problem is turned into solving a system of ordinary differential equations. Section 4 contains details on how the experiment was performed and we follow up with presentation and analysis of the computational results in Section 5. A simplified error estimate for the calculations is given in Section 6 and we close this text by a discussion and some concluding remarks that are entailed in Section 7.

2. Mathematical modelling

In this section we present the theoretical model used for analyzing our experimental data. The mathematical problem is as follows: determine the temperature distribution $T(x,t)$ for $0 \leqslant x < 1$ from measurements of temperature $f(t)$ and heat flux $h(t)$ along the line $x = 1$ where $T(x,t)$ satisfies:

$$\begin{cases} \dfrac{\partial}{\partial x}\left(\lambda(T)\dfrac{\partial T}{\partial x}\right) = \rho(T)\cdot c(T)\dfrac{\partial T}{\partial t}, & 0 < x < 1, t \geqslant 0, \\ T(1,t) = f(t), & t \geqslant 0, \\ \lambda(T)\cdot\dfrac{\partial T}{\partial x}(1,t) = h(t), & t \geqslant 0, \\ T(x,0) = 0, & 0 < x < 1. \end{cases} \tag{2}$$