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ScienceDirect

Applied Thermal Engineering 28 (2008) 380–387

APPLIED THERMAL ENGINEERING

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The inverse estimation of the thermal behavior and the viscosity of fluid between two horizontal concentric cylinders with rotating inner cylinder

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Received 8 February 2007; accepted 10 September 2007
Available online 14 September 2007

Abstract

This study is intended to provide an inverse approach for estimating the viscosity of fluid and thermal behavior of the concentric cylinders. Finite-difference methods are first employed to discredit the problem domain and then a linear inverse model is constructed to identify the condition for the viscosity of fluid and thermal behavior of the concentric cylinders. The present approach is to rearrange the matrix form of differential governing equation and to estimate coefficients of unknown conditions. Then, the linear least-squares error method is adopted to find the solution. Excluding measurement error, the inverse values from the present study are in excellent agreement with the direct solutions regardless of where the measurement points are located. When the measurement errors are considered, more measuring points are needed in order to increase the congruence of the estimated results to exact solutions. The advantage of applying this method in inverse analysis is that no prior information is needed on the functional form of the unknown quantities, and initial guess and iteration are not required.

Keywords: Concentric cylinders; Inverse; Linear least-squares method; Finite-difference methods

1. Introduction

The fluid flow and heat transfer in an annulus between two horizontal concentric cylinders have attracted considerable attention because of the theoretical interest and its wide technological applications such as cooling of rotating machinery, chemical vapor deposition processes, thermal energy storage systems, and transmission cables. Up to date, most of the work dealt with the direct natural convection problem [1–5] and mixed convection problem [6–10] in horizontal annulus. These studies focused on the flow and temperature pattern, and stability problem. However, to the authors' knowledge, the inverse heat transfer problem to deal with an annulus between two horizontal concentric cylinders problems has never been studied. In such cases, the inverse analysis of heat transfer can provide a powerful technique to estimate the unknown conditions. To date, analytical and numerical approaches have been developed to solve the inverse heat conduction problem. These techniques include the least-squares method with regularization [11–13], the sequential function specification method [11], space marching techniques [14–16], and the gradient method utilizing the adjoin problem [17]. Only little amount of work is available in the area of inverse analysis of convection inside ducts problem. Ozisik and Huang [18] used the regular and modified conjugate gradient methods to estimate the unknown steady-state distribution of wall heat flux for laminar forced convection inside a parallel-plates duct. Ramanujam [19] applied the quasi-Newton conjugate gradient method (which is a special case of conjugate gradient method) to obtain the temperature profile at the entrance of a thermally developing hydrodynamically developed laminar flow between parallel plates. Ozisik

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1359-4311/$ - see front matter © 2007 Published by Elsevier Ltd.
doi:10.1016/j.applthermaleng.2007.09.002

Nomenclature

A	matrix
B	the coefficient matrix of $\underline{\theta}$
c_p	the heat capacity of the fluid
e	the radial distance between inner cylinder and outer cylinder
k	the fluid heat conductivity
q_w	heat flux
R	the reverse matrix of the inverse problem
r_1	the radius of rotating inner cylinder
r_2	a radius of stationary outer cylinder
T	dimensionless temperature
μ	the dynamic viscosity of the fluid
Φ	viscous dissipation
ρ	the density of liquid
α	thermal diffusivity
σ	measurement error
$\underline{\underline{\boldsymbol{\theta}}}$	matrix
$\bar{\boldsymbol{\theta}}$	the coefficient vector of q, boundary conditions, and the dynamic viscosity of the fluid
ω	the angular velocity of the rotating inner cylinder

Subscripts

i	index of r-coordinate
j	index of θ-coordinate

and Bokar [20] estimated the unknown time-varying inlet temperature by regular and modified conjugate gradient methods. Recently, Ozisik and Liu [21] used the Levenberg–Marquardt method for the minimization procedure in the estimation of the thermal conductivity k and heat capacity c_p of the flow inside a circular duct by utilizing simulated transient temperature taken at a single location in the downstream region. However, the above methods are similar in essence, that must be chosen an arbitrary initial guess at iteration n to start the computation and repeat the calculation procedure for the next steps until the stopping criterion is satisfied, if not, return to initial step.

In this study, a different methodology [22,23] is presented. This method has been applied to many different applications. For instance, inverse laminar film condensation problems [24], inverse electronic device problems [25], a 3D inverse working roll problem [26], a 2D inverse laminar duct flow problem [27]. The method rearranges the matrix form of the direct problem in order to represent the unknown conditions explicitly. Then, the inverse model can be used to solve through the least square error method. The advantage of applying this method in inverse analysis is that no prior information is needed on the functional form of the unknown quantities, and initial guess and iteration are not required. Furthermore, the effects of sensor position, magnitude of measurement error and the number of measurements on the accuracy of estimates are examined.

2. Analysis

2.1. Description of the proposed model

Consider the two-dimensional geometry shown schematically in Fig. 1. The inner cylinder of radius r_1 rotates with a constant angular velocity ω within a stationary outer cylinder of radius r_2. Both the cylinders are assumed to be of infinite extent in the axial Z-direction and concentric. The radius of the cylinders is separated by a distance e.

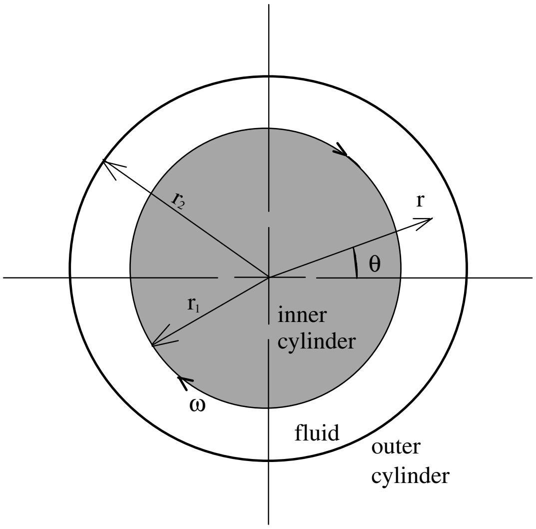

Fig. 1. Definition sketch of the cylinders.

The fluid flow between two cylinders is considered steady, incompressible, laminar and the streamlines to be circular. The momentum and energy equations in cylindrical coordinates of the problem are presented as:
Momentum equation

$$\frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r^2} = 0 \tag{1}$$

where v_θ is the velocity component of the fluid in the direction of the θ-axis.
Integrating twice, there results

$$v_\theta = \frac{A}{2}r + \frac{B}{r} \tag{2}$$

Applying the boundary conditions $v_\theta = r_1\omega$ at $r = r_1$ and $v_\theta = 0$ at $r = r_2$, we can find that

$$v_\theta = \frac{r_1^2\omega}{r_2^2 - r_1^2}\left(\frac{r_2^2}{r} - r\right) \tag{3}$$