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Fully developed natural convection heat and mass transfer in a vertical annular porous medium with asymmetric wall temperatures and concentrations

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Abstract

This work examines the effects of the modified Darcy number, the buoyancy ratio and the inner radius-gap ratio on the fully developed natural convection heat and mass transfer in a vertical annular non-Darcy porous medium with asymmetric wall temperatures and concentrations. The exact solutions for the important characteristics of fluid flow, heat transfer, and mass transfer are derived by using a non-Darcy flow model. The modified Darcy number is related to the flow resistance of the porous matrix. For the free convection heat and mass transfer in an annular duct filled with porous media, increasing the modified Darcy number tends to increase the volume flow rate, total heat rate added to the fluid, and the total species rate added to the fluid. Moreover, an increase in the buoyancy ratio or in the inner radius-gap ratio leads to an increase in the volume flow rate, the total heat rate added to the fluid, and the total species rate added to the fluid.

Keywords: Heat and mass transfer; Non-Darcy natural convection; Annular duct; Porous medium; Analytic solution

1. Introduction

Natural convection heat and mass transfer driven by combined thermal and solutal buoyancy forces in fluid-saturated porous media may be met in geophysical, geothermal and industrial applications. Natural convection heat and mass transfer for Newtonian fluids in porous media have been studied by many researchers [1–4]. Bejan and Khair [1] studied the heat and mass transfer by natural convection in a porous medium. Lateral mass flux effects and magnetic field effects on the natural convection heat and mass transfer along a vertical surface have been studied by Murthy and Singh [2] and by Cheng [3], respectively. They have shown that these two factors are very important for the free convection heat and mass transfer. Nield and Bejan [4] summarized the study on the phenomena of natural convection in porous media. Rastogi and Poulikakos [5] studied the natural convection heat and mass transfer from a vertical surface in a porous region saturated with a non-Newtonian fluid. They have shown that the non-Newtonian power-law index has significant effects on heat and mass transfer rates.

There are previous works that investigated free convection flow in the annular geometry filled with a porous material [6–9], which includes (a) free convection in a vertical cylindrical annuli by Havstad and Burns [6], (b) free convection in a horizontal space bounded by two concentric cylinders for Darcian fluid by Bejan and Tien [7] and for non-Darcy flow by Muralidhar and Kulacki [8], and (c) free convection through an vertical annulus with mixed boundary conditions by Jha [9]. Note that Jha [9] has shown that the Darcy number and the ratio of outer to inner radii are the governing factors for heat transfer through an annular porous medium.

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Nomenclature

C	concentration
D	mass diffusivity
Da	modified Darcy number
g	gravitational acceleration
Gr	Grashof number
H	total heat rate added to the fluid
K	permeability
m	wall temperature ratio
n	wall concentration ratio
N	buoyancy ratio
Nu	Nusselt number
Q	volume flow rate
Ra	Darcy-modified Rayleigh number
Sh	Sherwod number
T	temperature
u	velocity component
U	dimensionless velocity component
r	radial coordinate
R	dimensionless radial coordinate

Greek symbols

β_c	coefficient of concentration expansion
β_t	coefficient of thermal expansion
ε	porosity
θ	dimensionless temperature
λ	inner radius-gap ratio
ν	kinematic viscosity
ϕ	dimensionless concentration
Φ	dimensionless total species rate added to the fluid

Subscripts

0	condition at the inlet
1	condition at the outer surface of the inner cylinder
2	condition at the inner surface of the outer cylinder

In this work, we want to extend the work of Jha [9] to study the mass transfer effects on the fully-developed heat and mass transfer by natural convection inside a vertical annular duct filled with porous media for asymmetric wall temperatures and concentrations. The closed-form exact solutions are derived for the problem by using the non-Darcy flow model and the effects of the modified Darcy number, the buoyancy ratio and the inner radius-gap ratio on heat transfer and mass transfer are examined.

2. Analysis

Consider a steady fully developed laminar natural convection flow in annular region of infinite length embedded in a homogeneous fluid-saturated porous medium. The schematic diagram is shown in Fig. 1. The inlet temperature is T_0 and inlet concentration is C_0. The outer surface of the inner cylinder is kept at a constant temperature T_1 while the inner surface of the outer cylinder is maintained at a constant temperature T_2. In addition, the concentration of a certain constituent in the solution that saturates the porous medium varies from C_1 on outer surface of the inner cylinder to C_2 on the inner surface of the outer cylinder. Because the flow is fully developed, the flow depends only on the radial coordinate r. The fluid properties are assumed to be constant except for density variations in the buoyancy force term.

With introducing Boussinesq approximations, the equations governing the steady-state conservation of mass, momentum, energy and constituent for non-Darcy flow through a homogeneous porous medium inside the annular duct can be written as [9–11]

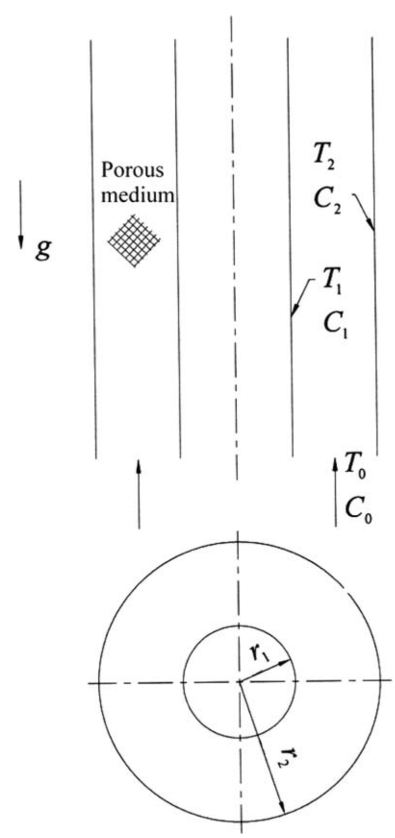

Fig. 1. The schematic diagram.

$$\frac{\nu}{\varepsilon}\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)-\frac{\nu}{K}u=-g\beta_t(T-T_0)-g\beta_c(C-C_0) \quad (1)$$