Available online at www.sciencedirect.com
ScienceDirect
China Particuology 5 (2007) 420–424
China PARTICUOLOGY
www.elsevier.com/locate/cpart

Analysis of liquid bridge between spherical particles

Fusheng Mu, Xubin Su *
College of Mechanical and Electrical Engineering, Central South University, Changsha 410083, China

Received 9 January 2007; accepted 29 April 2007

Abstract

A pair of central moving spherical particles connected by a pendular liquid bridge with interstitial Newtonian fluid is often encountered in particulate coalescence process. In this paper, by assuming perfect-wet condition, the effects of liquid volume and separation distance on static liquid bridge are analyzed, and the relation between rupture energy and liquid bridge volume is also studied. These points would be of significance in industrial processes related to adhesive particles.

Keywords: Surface-wet; Liquid bridge force; Rupture energy; Viscous force

1. Introduction

The investigation of wet particles has great significance in many fields, such as spray process (Wang, Yu, & Zhou, 2003). There are several adhesion mechanisms between surface-wet particles, such as electrostatic force, van der Waals force and liquid bridge force, the last being usually several orders larger than the others (Simons, 1996), that is, the liquid bridge force may well dominante all others (Johansen & Schæfer, 2001). The interstitial fluid between particles can be Newtonian or non-Newtonian. Based on the Reynolds lubrication theory, Lian and Huang et al. (Huang, Li, & Xu, 2004; Huang, Xu, & Lian, 2002; Lian, Adams, & Thornton, 1996; Lian, Thornton, & Adams, 1993; Xu, Huang, & Li, 2002; Xu, Huang, & Xu, 2005) studied the normal moving or tangential slipping between wet particles, and obtained expressions of the respective forces. Rossetti, Pepin, and Simons (2003) and Pitois, Moucheront, and Chateau (2001) explained the adhesion mechanism using rupture energy. In this paper, the acting mechanisms of liquid bridge and rupture energy are obtained by analysing the liquid bridge force, the viscous force and the rupture energy of a pair of central moving spherical particles connected by a pendular liquid bridge with interstitial Newtonian fluid.

2. Analysis of state of wet particles

2.1. Liquid bridge geometry

Fig. 1 shows two particles connected by a pendular liquid bridge which is supposed to be of a toroidal shape having a constant mean surface curvature for sufficiently small Bond numbers.

Since spherical agglomerates are usually formed from sub 250 μm particles and the liquid volume is quite small in many industrial processes, the bond number is very small, and the effect of gravity is negligible and the toroid assumption of the liquid bridge is valid (Rossetti & Simons, 2003).

In the Young–Laplace equation,

$$\Delta P = \gamma \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \quad (1)$$

where ΔP is the capillary suction pressure, γ the liquid surface tension, r_1 and r_2 are the two principal radii of curvature of the liquid bridge surface, while the former is negative and the latter positive.

The volume of liquid bridge is given by

$$V = \int_0^b 2\pi y H(y)\,\mathrm{d}y = \frac{\pi R}{2}[H^2(b) - D^2] \quad (2)$$

where D is the spherical surface distance, $H(b) = D + b^2/R$ and b is the radius of the wetted area. Relationships between liquid bridge volume and granularity, distance can thus be obtained.

* Corresponding author.
E-mail address: buchesssxb@163.com (X. Su).

1672-2515/$ – see inside back cover © 2007 Chinese Society of Particuology and Institute of Process Engineering, Chinese Academy of Sciences. Published by Elsevier B.V.All rights reserved.
doi:10.1016/j.cpart.2007.04.006

Nomenclature

b	radius of wetted area (m)
c	rupture energy constant
D	spherical surface distance (m)
D_{rupt}	static bridge rupture distance (m)
D^{d}_{rupt}	dynamic bridge rupture distance (m)
D^{*}	dimensionless separation distance
F_{cap}	liquid bridge force (N)
F_{vis}	viscous force (N)
m	constant
ΔP	capillary suction pressure (Pa)
r	principal radii of curvature of liquid bridge surface (m)
R	radius of particle (m)
v	separation speed (m/s)
V	liquid bridge volume (m^3)
V^{*}	dimensionless liquid bridge volume
W	rupture energy (J)
W^{*}	dimensionless rupture energy
Greek letters	
β	half-filling angle (degree)
γ	liquid surface tension (N/m)
η	dynamic viscosity (pa s)
θ	contact angle (degree)

2.2. Forces between particles

The liquid bridges may generate static liquid bridge force due to the curvature of the bridge and surface liquid tension forces, and dynamic forces due to the liquid viscosity as particles move relative to each other (Iveson, Litster, Hapgood, & Ennis, 2001). In this work, we only consider the dynamic liquid bridge forces for central moving particles and the interstitial fluid is Newtonian.

2.2.1. Liquid bridge force

There are two methods to calculate the liquid bridge force: Gorge method and boundary method (Iveson et al., 2001). Since the saturation is small and a neck will be formed when the bridge stretches, the Gorge method can be used for which the liquid bridge force is

$$F_{cap} = \pi \, \Delta P r_2^2 + 2\pi r_2 \gamma \tag{3}$$

According to geometry, $r_1 = (D/2 + R(1 - \cos\beta))/(\cos(\beta+\theta))$ and $r_2 = R\sin\beta - [1 - \sin(\beta+\theta)]\mathrm{r}_1$, and when $R \gg r_2 \gg r_1$ and $D \ll 2r_1\cos\theta$, a simplified expression can be written as

$$F_{cap} \approx 2\pi R\gamma\cos\theta\left[1 - \frac{D}{2r_1\cos\theta}\right], \tag{4}$$

Substituting Eq. (2) into Eq. (4), we get

$$F_{cap} = 2\pi\gamma R\cos\theta\left[1 - \left(1 + \frac{2V}{\pi D^2 R}\right)^{-1/2}\right], \tag{5}$$

Relationships between static liquid bridge force and liquid bridge volume, granularity, distance can then be obtained.

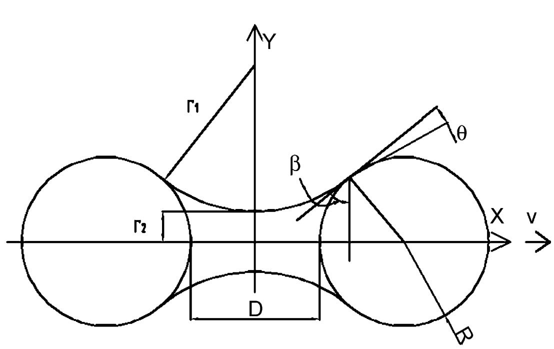

Fig. 1. A pair of central moving spherical particles with interstitial Newtonian fluid.

2.2.2. Viscous force

For Newtonian interstitial liquid, the Reynolds equation is

$$\frac{\mathrm{d}}{\mathrm{d}y}\left[yH^3(y)\frac{\mathrm{d}P(y)}{\mathrm{d}y}\right] = 12\eta y\frac{\mathrm{d}D}{\mathrm{d}t}, \tag{6}$$

where η is the dynamic viscosity of the liquid. By integrating Eq. (6) twice, an expression can be derived for the viscous force acting on the spheres:

$$F_{vis} = -\frac{3}{2}\pi\eta R^2\frac{1}{D}\frac{\mathrm{d}D}{\mathrm{d}t}, \tag{7}$$

Note that Eq. (7) can be used only under the condition of infinite liquid. In the case of finite volume of liquid, Mattheuson (Pitois, Moucheront, & Chateau, 2000) introduced a correction factor to Eq. (7):

$$F_{vis} = -\frac{3}{2}\pi\eta R^2\left[1 - \frac{D}{H(b)}\right]^2\frac{1}{D}\frac{\mathrm{d}D}{\mathrm{d}t}, \tag{8}$$

Relationships between dynamic liquid bridge force due to fluid viscosity and granularity, distance can then be obtained.

2.2.3. Rupture energy

It is important to determine whether particles will adhere or rebound when colliding with each other, in order to investigate the mechanism of adhesion and agglomeration. Simons and co-workers (Rossetti et al., 2003) developed a model to provide an approximate value of the rupture energy of pendular liquid bridges. They found that if the relative kinetic energy of the colliding particles was below the work required to break the liquid bridge then the particles would adhere together. A simple expression was derived in terms of dimensionless parameters as follows:

$$W^* = cV^{*m}, \tag{9}$$

where c is the rupture energy constant and m is the power index and the dimensionless parameters follow:

$$W^* = \frac{W}{\gamma R^2} \quad \text{and} \quad V^* = \frac{V}{R^3}$$