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Study of falling water drop in stagnant air

M. Abdelouahab [a,b,*], R. Gatignol [c]

[a] Université des Sciences et de la Technologie, Mohammed Boudiaf, BP 1505, Oran, Algérie

[b] Laboratoire de Modélisation des Systèmes Industriels, U.S.T.O, Algérie

[c] Sorbonne Université UPMC (Paris 6) & CNRS, Institut Jean le Rond d'Alembert, 4 place Jussieu, 75005, Paris, France

HIGHLIGHTS

- The parameters studied are the instantaneous speed versus time and as a function of the height of fall for small drops and extend to large drops.
- An equation relating the equivalent diameter to the terminal height of falling drop reaching 99% of the terminal velocity is proposed.
- The results obtained are comparable to the experimental data extracted from the literature.

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ABSTRACT

The objective of this work is to propose a *mathematical* expression for the instantaneous velocity of falling water droplets in air and traveled height versus time. In addition, changes in terminal velocity depending on terminal height are studied. For very small spherical drops, having very low Reynolds number and obeying Stokes law for the drag force, the instantaneous velocity u of the drop with time is a well-known result. From the expression of that speed, it is easy to get the drop height h versus time. By eliminating the time, the displacement h of the droplet as a function of the velocity u is obtained. In the present work, the latter was extended to larger drops. *The values predicted by the mathematical equation obtained from the combination of theory and experiment (semi empirical equation) are compared with well known experimental data*. The results show good agreement between the values predicted and the experimental data.

1. Introduction

Mechanics demonstrate that the rate of fall of a body in a continuous medium tends asymptotically towards a limit speed. Theoretically, a drop in air takes an infinite time to reach its terminal velocity. Practically, it is assumed that this speed is reached when it reaches 95% of the terminal velocity. In this work, we propose a formula describing the motion of a drop up to 99% of its terminal velocity and compare to other authors who made measurements at the same percentage.

Researchers generally introduce the equivalent diameter of the drop (d_e), the diameter of the spherical drop having the same volume as the deformed drop. To study the rate of fall of water droplets in stagnant air there are three categories: (1) For very small drops ($d_e < 80$ μm), the speed can be calculated using the Stokes drag formula; (2) for the average drops (80 μm $< d_e <$ 500 μm), the speed can be calculated using the dynamics law of Newton with the determined drag coefficient for solid sphere; (3) for the large drops (500 μm $< d_e <$ 5 mm) which are deformed due to air friction and instead of maintaining a spherical shape, flatten, thus presenting a larger surface area to the air in which they fall. These drops become unstable and explode when their diameter exceeds 5 mm. Many studies in this field have been undertaken but less successfully. The experimental data we have analyzed concern the falling drops of water in the air: Lenard [1], Flower [2], Laws [3], Gunn and Kinzer [4], Leeden et al. [5], Sartor and Abbott [6], Wang and Pruppacher [7], Beard [8], Boxel [9], Andsager et al. [10], Zhou et al. [11] and Chowdhury et al. [12]. Often rainy fall simulators are used for the study of the process, but most are not high enough for large accelerated drops to reach their terminal velocity. Most simulators have a height of 2 m and many others have a smaller height.

In their experiments, Wang and Pruppacher [7] used a tower that extends vertically over nine floors of a building in 1968. It measures 35 m in height and has an area section of about 1 m^2. For very small water droplets that remain spherical having the

* Corresponding author at: Université des Sciences et de la Technologie, Mohammed Boudiaf BP 1505, Oran, Algérie.

E-mail addresses: abdelouahab.moha@gmail.com (M. Abdelouahab), renee.gatignol@upmc.fr (R. Gatignol).

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Reynolds number much less than 1, it is easy, after the resolution of the equation of motion, to obtain the well known result of the fall speed versus height (see paragraph 2) where the results are very encouraging. By moving to larger drops, having large density ratio, as in our case it is of the order of 1000, the study of the movement of the droplet becomes difficult.

Recently another work on the same subject was made by Chowdhury et al. [12]. They investigate the shape "axial ratio" and the falling speed of the water drops in stagnant air kept at constant temperature to 22 °C. To carry out this project, they use a tower high more than 13 m. Experiments were carried out for three different sizes of falling drops (0.26, 0.37 and 0.51 cm equivalent diameter) for the fall distance of up to 13 m. Based on experimental observations, they divided the path traveled by the drop in three distinct zones (Zone I, Zone II and Zone III). In Zone I, the shape of the drop undergoes continuous adjustment due to horizontal and vertical oscillations and the progressive viscous damping which are the main features. Zone II is characterized by drops of constant shape, called equilibrium form, falling without oscillations. In this area the drops continue to accelerate to the terminal velocity until the end of it. In zone III, the drops have already gained the equilibrium shape and finish the trip with a speed equal to 99% of the terminal velocity. Necessary fall distances are found to be smaller than the distance described in the literature. Based on experimental results, Chowdhury et al. [12] conclude that the fall height values of approximately 6 and 12 m can be used as reference data for rainfall experiments.

Previous theoretical and experimental works have been done but none has a general character. Indeed, experimentally, we know only the work of the authors cited above and from studies using sophisticated methods such as, for example, the resolution of the Navier–Stokes equations both in the droplet and surrounding fluid. We cite for example the work of Gottesdiener et al. [13], in which the numerical simulation was based on the finite volume method and the equation of motion was solved. Each numerical method is related to the assumptions and conditions which may vary depending on the model. Indeed, some solutions are valid for low Reynolds numbers, while other valid only for very small drops or for some choice of the domain or mesh or boundary conditions. The proposed solutions are local and cannot be generalized.

We proposed a very simple idea. We begin by studying the movement of small water droplets that remain spherical in the air at rest, having Reynolds number less than 1 (Section 2.1). The experimental data used for comparison are those of Sartor and Abbott [6]. This has already been studied by Wang and Pruppacher [7]. The advantage here is that the drag coefficient is known and very simple formulation, and the differential equation obtained can be solved analytically.

Then (Section 2.2), this solution is generalized to larger drops which deform. Expressions of drag coefficient are complicated and the differential equation obtained for the movement is impossible to solve analytically. A model is proposed to express the drop height depending on the instantaneous velocity and terminal velocity (Section 3). Indeed, it has a general character, and in addition, the theoretical values are comparable to the experimental data that exist in the literature. The range of validity of this solution is discussed (Section 4).

2. Presentation of the method

The vertical movement of a spherical drop in air at rest is described by the fundamental equation of dynamics which is inserted, in particular, the historical term of Basset–Boussinesq (Clift et al. [14]). In the case of particles whose density is very high compared to that of the continuous phase, this historical term can be neglected. This applies very well in the case of droplets dispersed in the air where the ratio of the densities is very important. In addition, the effects of the Basset–Boussinesq force are even lower than the viscosity of the surrounding medium is important (Clift et al. [14]).

Ignoring the historical term in the fundamental equation of dynamics, the following equation is used to a falling drop in air:

$$(\rho_d + \epsilon \rho_c)\,\frac{du}{dt} = (\rho_d - \rho_c)g - \rho_c \frac{S}{V} C_D \frac{u^2}{2} \tag{1}$$

where $\epsilon = \frac{1}{2}$ for a sphere $\rho_c\ V$ is the added mass C_D the drag coefficient and S the projection of the surface of the drop on the plane perpendicular to the direction of movement; u, V and ρ_d represent the speed, volume and density of the drop and ρ_c is the density of the continuous phase. Indices c and d are respectively for the continuous phase and the dispersed phase.

2.1. For small spherical drops ($d_e < 160$ µm)

For a spherical particle in uniform motion with the velocity u in an infinite liquid at rest with a low Reynolds number, Hadamard [15] and Ribczynski [16] propose the following drag coefficient:

$$C_D = \frac{8\ (2 + 3\kappa)}{Re\ (1 + \kappa)} \tag{2}$$

where Re is the Reynolds number of a liquid particle in an infinite medium at rest and $\kappa = \frac{\mu_d}{\mu_c}$ viscosity ratio. The Reynolds number is defined by:

$$Re = \frac{\rho_c\ u\ d_e}{\mu_c}. \tag{3}$$

For a solid sphere or a drop of $\kappa \to \infty$, Eq. (2) gives:

$$C_D = \frac{24}{Re}. \tag{4}$$

Eq. (4) is valid for $Re < 0.1$. The accuracy of this relationship is 1% in the range of the Reynolds number indicated. It can be used even for $Re > 0.1$ but with error of 10%.

For rising bubbles, $\kappa \to 0$ and Eq. (2) leads to:

$$C_D = \frac{16}{Re}. \tag{5}$$

Considering the particle spherical and the added mass negligible (because $\epsilon \rho_c$ product is negligible compared to ρ_d), using Eqs. (2) and (3) (assuming a quasi-static nature), then Eq. (1) becomes:

$$\frac{du}{dt} = \frac{(\rho_d - \rho_c)}{\rho_d} g - \frac{6\mu_c (2 + 3\kappa)}{\rho_d\ d_e^2\ (1 + \kappa)}\ u. \tag{6}$$

By taking $K_1 = \frac{g \Delta\rho}{\rho_d}$ and $K_2 = \frac{6\mu_c (2+3\kappa)}{\rho_d\ d_e^2\ (1+\kappa)}$, one obtains:

$$\frac{du}{dt} = K_1 - K_2\ u. \tag{7}$$

The resolution of Eq. (7) gives:

$$u = U_T\ \left(1 - e^{-K_2\ t}\right) \tag{8}$$

where $U_T = \frac{K_1}{K_2}$. U_T is the terminal velocity of Stokes. It is:

$$U_T = \frac{\Delta\rho\ g\ d_e^2\ (1 + \kappa)}{6\mu_c\ (2 + 3\kappa)}. \tag{9}$$

When $\kappa \to \infty$, Eq. (9) approaches the terminal velocity of Stokes drops and solid spheres.

$$U_T = \frac{\Delta\rho\ g\ d_e^2}{18\ \mu_c}. \tag{10}$$