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Direct numerical simulation of the subcritical electroconvective instability in a dielectric liquid subjected to strong or weak unipolar injection

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ABSTRACT

This work addresses the characterization of the subcritical electroconvection instability that occurs in a plane layer of a dielectric liquid subjected to both strong and weak unipolar injections. Transient evolutions of electrohydrodynamic convection in a dielectric liquid between two parallel plates are analysed numerically to best determine the linear and non linear instability criteria, T_c and T_f, respectively. In strong injection as well as in weak injection the linear stability parameters T_c that we obtained by direct numerical simulations are very close to the ones predicted by the stability analysis. However, in both strong and weak injections, it is shown that the non linear criterion T_f provided by the stability analysis or the available analytical models differs from the one obtained from direct numerical simulation. These discrepancies are analysed and explained. In particular, in the case of weak injection the inertial terms play an important role in the development of the flow structure. This is reflected in a dependence of T_f on M, a dependence that has not been modelled by any analytical o semi-analytical approach, of the type of Felici hydraulic model. In this paper we put a particular emphasize on numerical computations in weak injection. We have successfully made computations for small values of the mobility parameter M in weak injection, computations that demand a very efficient numerical code. For $M = 5$ and $M = 10$ a steady convective regime has been captured. In the vicinity of $M = 15$ the flow exhibits a perfect periodic regime with a well defined fundamental frequency while for M above 20 the flow is unsteady and turns out to be chaotic for higher values.

1. Introduction

Electrohydrodynamics (EHD) is a multidisciplinary area dealing with the interaction of fluids and electric field [1] and many industrial processes as well as natural situations are concerned with this very active topic [2]. When metallic electrodes (one of them being at a high potential and the other earthed) are immersed in a dielectric liquid of low enough conductivity injection of electric charges occurs. The electrochemical mechanisms giving birth to the injection of charges at the electrodes have been studied in depth by Felici [3]. It can take place at both electrodes or at only one of them. In this paper we consider that injection only occurs at one electrode (unipolar injection) only. The Coulomb force acting upon the injected space charges will put the fluid into motion. In gases, the charges follow the electric field lines and move much faster than the fluid and the distribution of charge is weakly coupled to the fluid motion. However, in liquids the fluid velocity is dominant and a strong coupling between the hydrodynamic and the electrical effects arises. Indeed the dynamics of the liquid is greatly influenced by the electric Coulomb force exerted upon the injected space charge, while the space charge distribution is determined by the velocity and the electric fields.

Several situations can be considered, depending on the geometry and shape of the electrodes. In this paper we focus on a typical configuration, which has been widely studied: two plane and parallel electrodes immersed in a dielectric liquid [1–10]. In this particular configuration when the applied voltage is high enough the Coulomb force gives rise to the development of an instability that puts the liquid into motion. In this classical EHD problem two typical regimes are usually considered: strong or weak injection, depending on the value of a non-dimensional parameter C, which measures the injection strength. The authors who have been

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interested so far in EHD flows have used different mathematical techniques. Stability analysis has been conducted and a good review of the linear and non-linear stability regimes can be found in [4–9]. Several experimental studies have also been carried out but mainly in the strong injection regimes [10–12]. As a matter of fact, this strong and non-linear coupling as well as the complexity of the mathematical problem has prevented the obtainment of any analytical solution. This inherent complexity has encouraged the use of numerical methods to gain additional insight into physical phenomena. The first attempt to solve the whole coupled system of partial differential equations was conducted by Castellanos et al. [7,8]. The authors solved the Navier–Stokes equations using the finite difference method on a staggered grid and the SIMPLER [13] algorithm. The charge density transport equation was also solved using the finite difference method and the Upwind Differencing scheme for the convective term.

However, this numerical method was not able to solve accurately the charge density transport equation. Indeed, since this equation is hyperbolic, it requires the use of specific numerical methods to deal with the very steep gradients that may arise in the charge distribution. To tackle this problem they developed an original Particle In Cell (PIC) method which consists in injecting numerically charged particles in the bulk. This technique was resumed latter by Chicón, Castellanos and Martín [14]. Vázquez, Georghiou and Castellanos [15], compared the particle in cell method with the integration of the charge density transport equation based on the Flux Corrected Transport (FCT) scheme [16]. However in these papers, the fluid velocity field is not computed from the Navier–Stokes equations. Instead, the authors assumed that the flow adopts the structure of one convective roll and the velocity field is deduced from an analytical expression given by the stream function associated with this roll structure. The main drawback of such approach, referred as Imposed Velocity Field (IVF), is that the velocity is not computed from the governing equations. The relevance of this approach is therefore strongly limited to some situations where the assumptions used to express analytically the velocity field remain valid. This is the case only in strong injection and for values of the electrical Rayleigh number close to the critical instability threshold [17].

Later Vázquez, Georghiou and Castellanos [18] performed a whole integration of the Navier–Stokes equations by a fractional-step finite element method connected with the particle in cell method or the FCT based one.

In [19] Traoré et al. successfully solved the full and complete set of equations associated with the electro-thermo-convective phenomena that take place in a planar layer of dielectric liquid heated from below and subjected to unipolar injection. In [20] the authors analysed the flow structure induced by a strong injection as well as the route to chaos. They used a finite volume method to discretize the partial derivative set of equations. The hyperbolic charge density transport equation was accurately solved using a Total Variation Diminishing (TVD) scheme.

Very recently Vázquez and Castellanos [21] obtained some interesting results using a Discontinuous Galerkin Finite Element based Method (DG-FEM). According to the authors DG-FEM is especially suited to solve purely hyperbolic equations such as the charge density transport equation. In strong injection regime these authors found a non-linear criterion value $T_f = 108.7$ very close to the one obtained by Traoré et al. [20] but quite far from the theoretical value 125 predicted by the nonlinear stability analysis. This discrepancy has never been explained nor justified.

This work addresses the stability of a plane layer of a dielectric liquid subjected to unipolar injection. Both strong and weak injection regimes are considered for a wide range of the M parameter. Results from all previous numerical studies have been summarized and compared. We particularly emphasize the weak injection regime, especially for small M values through the flow structure and its behaviour.

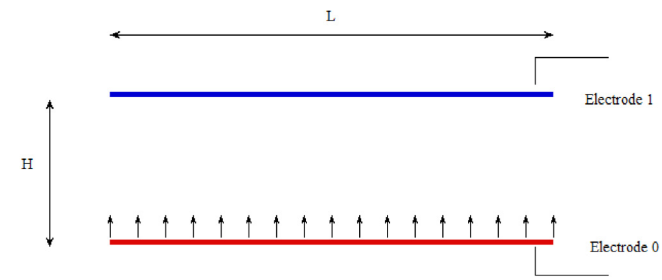

Fig. 1. Sketch of the physical domain.

The reminder of this paper is organized as follows. In the following section the problem is stated and the governing equations and boundary conditions are described. Then in Section 3 some hints and tips are given concerning the numerical procedure and especially on the treatment of the charge density equation which is the key point. In Section 4 the procedure to determine the instability thresholds is detailed. Our numerical results are discussed in Section 5, and compared with the ones that several authors have previously obtained. Finally a conclusion is drawn up in Section 6.

2. Statement of the problem

2.1. Governing equations

The system under consideration is a dielectric liquid layer of width H enclosed between two electrodes of length L (Fig. 1). The layer is subjected to a potential difference $\Delta V = V_0 - V_1$ which produces a perpendicular electric field $\vec{E}$. Charge is injected into the bulk from the emitter electrode.

The fluid of density ρ_0, dynamic viscosity η and permittivity ε, is assumed to be incompressible and perfectly insulating. The complete formulation of the flow of dielectric liquids subjected to electric field is governed by the Electro-Hydro-Dynamics (EHD) equations [22,23].

$$\nabla \cdot \vec{u} = 0 \tag{1}$$

$$\rho_0 \left(\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} \right) = -\nabla \tilde{p} + \eta \Delta \vec{u} + q\vec{E} \tag{2}$$

$$\frac{\partial q}{\partial t} + \nabla \cdot \vec{j} = 0 \tag{3}$$

$$\Delta V = -\frac{q}{\varepsilon} \tag{4}$$

$$\vec{E} = -\nabla V \tag{5}$$

where $\vec{u}$ is the fluid velocity, $\tilde{p}$ the modified pressure which includes the pressure and the scalar from which the electrostriction force derives. As the fluid is assumed to be homogeneous and isothermal the dielectric force vanishes and only the Coulomb force $q\vec{E}$ acts on the fluid. q is the charge density and $\vec{j}$ is the electrical current density.

We assume that the injection is *unipolar* which means that the charge carriers are considered to be of the same type with an ionic mobility K so they migrate along the liquid with a velocity $K\vec{E}$. We further assume a *homogeneous* and *autonomous* injection. That means that the density of charge at the emitter electrode, assumed to be at $z = 0$, is constant in time and equal to q_0. The injector and hence the injection rate are neither influenced by the electric field nor by the liquid motion [23]. It is also considered that the ions discharge instantaneously once they reach the collector electrode [24].