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Motion of a viscous droplet bisecting a free surface of a semi-infinite micropolar fluid

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ABSTRACT

The Stokesian flow of a spherical-shaped droplet which is halfway immersed in a semi-infinite phase of a micropolar fluid is discussed, the surface of which is assumed to remain flat. This configuration is studied analytically using the stream function formulation in two different settings, when the movement of the droplet perpendicular to the free flat surface of the micropolar fluid and the parallel motion. The interface conditions on the droplet boundary are that the velocity is continuous, the shear stress is continuous, and the microrotation is proportional to the vorticity. Analytical solutions for the stream functions outside and inside the droplet are obtained in each case of the droplet movement. The drag force acting, in each case, on the part of fluid sphere immersed in the micropolar fluid is evaluated. Numerical results for the drag force coefficient versus the relative viscosity, micropolarity parameter and spin parameter are presented both in tabular and graphical forms. The results for the drag coefficient are compared with the available solutions in the literature for the limiting cases.

1. Introduction

Eringen [1] formulated the theory of micropolar fluids which display the effects of local rotary inertia and couple stresses. This theory can be used to explain the flow of colloidal fluids, liquid crystals, animal blood, polymer fluids, fluid suspension, etc. Physically, micropolar fluids may be described as non-Newtonian fluids consisting of dumb-bell molecules or short rigid cylindrical elements. The presence of dust or smoke, particularly in a gas, may also be modeled using micropolar fluid dynamics. This theory is capable of describing such fluids. In micropolar fluids, individual particles can rotate independently from the rotation and movement of the fluid as whole. Therefore, a new variable which represents the angular velocity of fluid particles and a new equation governing this variable should be added to the conventional model. Extensive review of micropolar fluid theory and some of its applications can be found in recent books [2,3] and references therein.

The motion of fluid droplets over fluid and solid surfaces has attracted the interest of many investigators in the past because of its numerous practical and industrial applications. It is well-known that the application of a body force or external gradients can be used as a mechanism for driving the motion of fluid drops, and the ability to control these properties can play a key role in many industrial processes such as coating and microfluidic devices [4–7]. The pioneer Stokes' flow problem of the steady-state translational motion of a spherical drop in an immiscible fluid was treated by Hadamard [8] and Rybczynski [9]. Hetsroni and Haber [10] used the method of reflection to solve the problem of a single droplet submerged in an unbounded viscous fluid of different viscosity. O'Neill et al. [11] have studied the motion of a solid spherical particle relative to a planar interface separating two immiscible incompressible fluids of widely disparate viscosities, while allowing for homogeneous Navier slip condition over the entire submerged sphere surface. Recently, Lee and Keh [12,13] investigated the slow translational and rotational motions of a spherical particle and fluid drop within a non-concentric spherical cavity that had slip surfaces, perpendicular to the line of their centers. Ramkissoon [14,15] has obtained the solution for Stokes' flow problem of a micropolar fluid flow around a Newtonian fluid sphere and spheroid. Niefer and Kaloni [16] discussed the two related problems of the flow of a viscous fluid past a fluid sphere which has a micropolar fluid inside it and the flow of a micropolar fluid past a viscous fluid drop with non-zero spin boundary condition. Hayakawa [17] solved the problems of axisymmetric slow viscous flow of a micropolar fluid past a stationary sphere and a stationary cylinder explicitly, and computed the drag force in each case. The resistance force exerted on a solid sphere moving with constant velocity in a micropolar fluid with a nonhomogeneous boundary condition for

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the microrotation vector was calculated by Hoffmann et al. [18]. Stokes flow motions of a sphere bisected by a free flat surface bounding a semi-infinite micropolar fluid have extensively been examined by Saad [19]. The problem of Stokes axisymmetrical flow of an incompressible micropolar fluid past a fluid droplet-in-cell models has been investigated analytically by [20], and the Stokes axisymmetrical flow caused by a viscous spherical droplet translating in a micropolar fluid perpendicular to a plane wall at an arbitrary position from the wall is presented in [21] as well as for the related problem of a micropolar fluid sphere immersed in a viscous fluid.

It should be noted here that there is no uniform consensus on the microrotation boundary conditions for micropolar fluids. An interesting review for various types of microrotation boundary conditions is given by Migun [22]. An important physically acceptable dynamic boundary condition for microrotation was suggested by Aero et al. [23] which states that the microrotation is proportional to the couple stress at the boundary. Also a spin–vorticity kinematic boundary condition was proposed by Condiff and Dahler [24] which states that the microrotation is proportional to the vorticity at the boundary.

The basic physical problem presented in this study relates to the hydrodynamic mechanisms permitting a droplet molecule to cross an interfacial region during an interphase mass transfer from one bulk-fluid solvent phase to another under the influence of a chemical potential difference [11]. This motivated us to study Stokes flow arising from the axisymmetric and asymmetric translational motions of a viscous fluid sphere bisected by a free surface bounding a semi-infinite micropolar fluid, which is assumed to be flat. Surface tension acting at the interface between the two immiscible fluids tends to keep the spherical shape of the fluid particle against the shearing stresses which tend to deform it. If the motion is sufficiently slow or the droplet sufficiently small size, the droplet will be spherical, at least in the first approximation [25]. Taylor and Acrivos [26] treated the problem of distortion which occurs when the inertial effects are no longer negligible for a viscous drop in a second immiscible unbounded viscous fluid. The floating fluid droplets can be used as containers for encapsulating reagents in biochemical reactions. They allow low consumption of the reagents and give direct access to reaction products [27]. Taking advantage of the noncoalescence phenomenon of a droplet on a liquid substrate, the researchers focused on manipulating droplets on the liquid free surface both experimentally and theoretically [28–31]. Shabani et al. [31] developed a data represented by non-dimensional groups of parameters could be used as a guideline to design experiments to form various sizes of floating droplets for the effective droplet manipulation. Greco and Grigoriev[32] investigated the problem of a droplet suspended at the interface between a substrate fluid and a covering fluid. They declared, under certain assumptions, the surface tensions at the upper surface of the droplet and at the substrate control the degree of submersion and the shape of the droplet. They also showed that, if the surface tension at the substrate vanishes then the droplet has a spherical shape. Therefore in this model, we assume that the deformation of the fluid particle is neglected and the particle keeps its spherical shape permanently. We confine our attention to the special case where the sphere is bisected by the plane of the interface, and to situations for which the contact angle is 90°. As boundary conditions, continuity of velocity, continuity of shear stress and the spin–vorticity relation at the droplet surface are used. Analytical solutions are obtained in each case for the stream functions and microrotation components. The drag acting (for axisymmetric and asymmetric cases) on the fluid droplet is evaluated. Numerical results for the drag force coefficient versus the relative viscosity, micropolarity parameter and spin parameter are presented both in tabular and graphical forms. The results for the drag coefficient are compared with the available solutions in the literature for the limiting cases.

2. Field equations

The equations governing the steady flow of an incompressible micropolar fluid under Stokesian assumption in the absence of body force and body couples are given by [3]

$$\operatorname{div}\vec{q} = 0, \tag{2.1}$$

$$\operatorname{grad} p + (\mu + k)\operatorname{curl}\operatorname{curl}\vec{q} - k\operatorname{curl}\vec{\nu} = 0, \tag{2.2}$$

$$k\operatorname{curl}\vec{q} - 2k\vec{\nu} - \gamma\operatorname{curl}\operatorname{curl}\vec{\nu} + (\alpha + \beta + \gamma)\operatorname{grad}\operatorname{div}\vec{\nu} = 0, \tag{2.3}$$

where $\vec{q}$, $\vec{\nu}$ and p are the velocity vector, microrotation vector and the fluid pressure at any point, respectively. μ is the viscosity coefficient of the classical viscous fluid and k is the vortex viscosity coefficient. The remaining constants α, β and γ are gyroviscosity coefficients.

The equations for the stress tensor t_{ij} and the couple stress tensor m_{ij} are given by the following constitutive relations

$$t_{ij} = -p\,\delta_{ij} + \mu\,(q_{i,j} + q_{j,i}) + k\,(q_{j,i} - \epsilon_{ijl}\,\nu_l), \tag{2.4}$$

$$m_{ij} = \alpha\,\nu_{l,l}\,\delta_{ij} + \beta\,\nu_{i,j} + \gamma\,\nu_{j,i}, \tag{2.5}$$

where the comma denotes partial differentiation with respect to the spatial coordinates, δ_{ij} and ϵ_{ijl} are the Kronecker delta and the alternating tensor, respectively.

3. Formulation of the problem

Stokes' flow motion of a viscous spherical droplet relative to a planar interface separating two nonmixing fluids is investigated as shown in Fig. 1. In general the fluids are assumed to be micropolar of widely different viscosities. Our aim in this work is to illustrate in a relatively simple manner how the drop assumption helps us to remove the contact-line singularity. These simplifications allow us not to consider an arbitrary degree of submersion of the droplet. We limit our investigation to the case where the viscosity ratios characterizing two fluids phases are large. In this case the planar surface may be considered as effectively stress and couple stresses free, and only the hydrodynamic forces exerted on that half of the spherical droplet immersed in the more viscous fluid need be considered in the calculations.

Consider a translating viscous fluid sphere S, of radius a and viscosity $\tilde{\mu}$, at the instant of time that it is exactly half-immersed in a planar free surface F bounding a semi-infinite micropolar fluid region D. The free surface is assumed to have no motion normal to itself. Instantaneously, we shall assume that the quasisteady motion (both axisymmetric and non-axisymmetric motions) of a spherical droplet translating with a constant velocity $\vec{U}$ in a second, immiscible micropolar fluid of viscosities $(\mu, k, \alpha, \beta, \gamma)$. The velocity vector may be arbitrarily oriented relative to the normal to the free flat surface F. Let (x, y, z), (ρ, ϕ, z) and (r, θ, ϕ) denote the system of rectangular Cartesian axes, circular cylindrical and spherical coordinate systems, respectively, centered at O. The z-axis lies normal to the free surface and points into the region of the micropolar fluid (see Fig. 1).

We define the following three regions in terms of this spherical coordinate as follows:

(i) The semi-infinite micropolar fluid region D bounded by the free flat surface

$$r > a, \qquad 0 \le \theta < \frac{\pi}{2}, \qquad 0 \le \phi < 2\pi. \tag{3.1}$$

(ii) The immersed fluid half-sphere S of the sphere surface

$$r = a, \qquad 0 \le \theta < \frac{\pi}{2}, \qquad 0 \le \phi < 2\pi. \tag{3.2}$$