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Creeping motion of a micropolar fluid between two sinusoidal corrugated plates

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ABSTRACT

The steady creeping motion of an incompressible micropolar fluid between two slightly corrugated plates is investigated using the perturbation technique up to the second order. Both normal and parallel flows to the corrugations are considered. The corrugations of the two walls are assumed to be either in phase or half-period out of phase. It is also, assumed that the corrugations are periodic sinusoidal waves of small amplitude. The assumption of low Reynolds number is applied so that the nonlinear inertial terms can be ignored. The effect of corrugations, the rate of flow and the mean velocity are illustrated versus the micropolarity, phase difference and wavelength of corrugations.

1. Introduction

The theory of micropolar fluids was proposed by Eringen to describe the correct behavior of a type of fluids with substructures. The motion of these fluids can be investigated by two vectors; the classical velocity vector characterizing the motion of macro-volume element and the microrotation vector which represent the rotation of microelements about their centroids [1]. This theory can be applied in an increasingly significant number of cases in various scientific fields. Some of these fields are the study of lubricating fluids in bearings in lubrication theory [2] and the physics of liquid crystals [3]. Ferrofluid can be modeled as a micropolar fluid; because it consists of a stabilized colloidal suspension of Brownian magnetic particles in a non-magnetic liquid host [4]. The granular flows have micro-structure and rotation of particles. So, the model of micropolar fluids can describe granular fluid flows correctly [5–8]. Hayakawa [5] has reported that the theoretical calculations of certain boundary value problems are in agreement with relevant experimental results of granular flows. The theory of micropolar fluids has attracted the attention of many authors e.g. [9–13].

The fluid flows through channels with corrugated or irregular boundaries have received considerable attention from researchers because of their wide range applications. Physically, studying fluid flow through channels with irregular surfaces has various applications in the fields of distillation columns and separation processes [14,15], biological transport phenomena and chemical separation [16,17] and determining pressure drop in microchannels with rough surfaces [18–25]. The effect of boundary irregularities can be characterized experimentally using, for example, optical measurement, scanning electron microscope, atomic force microscope and scanning tunneling microscope [26,16]. One of the important applications of irregular boundaries is the blood flow in the circularity system which usually encounters boundary irregularities in diseased vessels [17,27]. This results in the existence of abnormal flow conditions. An excellent description of such abnormalities is given by Chow and Soda [28]. Sanyal and Sarkar [29] investigated a steady wavy incompressible Newtonian fluid flow in a channel with irregular surfaces and used a haemodynamical solution to determine the effects of the wall roughness upon the blood oxygenation in a membrane oxygenator. Recently, the flow through corrugated channels has been studied by a number of researchers e.g. [30–35].

In the present work, we extend the work of Wang [33,34] to the theory of micropolar fluids. A perturbation technique is used to the second order for slightly corrugated walls [36]. Both transverse and longitudinal fluid flows are considered. The no-slip and no-spin boundary conditions are applied. The assumption of low Reynolds number is utilized so that the nonlinear inertial terms can be ignored.

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2. Field equations

The field equations governing an incompressible steady micropolar fluid flow in the absence of body forces and body couples are given by

$$div\,\vec{q}^{\,*} = 0, \tag{2.1}$$

$$-(\mu+\kappa)\ curl\ curl\,\vec{q}^{\,*} + \kappa\ curl\,\vec{\nu}^{\,*} - grad\ p^* = 0, \tag{2.2}$$

$$(\alpha+\beta+\gamma)\ grad\ div\,\vec{\nu}^{\,*} - \gamma\ curl\ curl\,\vec{\nu}^{\,*} + \kappa\ curl\,\vec{q}^{\,*} - 2\kappa\,\vec{\nu}^{\,*} = 0, \tag{2.3}$$

where $\vec{q}^{\,*}$, $\vec{\nu}^{\,*}$ and p^* are the velocity vector, microrotation vector and fluid pressure at any point. The material constants (μ,κ) are viscosity coefficients and (α,β,γ) are gyro-viscosity coefficients. These material constants are satisfying the following inequalities [1]

$$\begin{aligned} &\kappa \ge 0, \quad 2\mu+\kappa \ge 0, \quad \gamma \ge 0, \\ &\gamma \ge |\beta|, \quad 3\alpha+\beta+\gamma \ge 0. \end{aligned} \tag{2.4}$$

The constitutive equations for the stresses τ^*_{ij} and couple stresses m^*_{ij} have the forms

$$\tau^*_{ij} = -p^*\,\delta_{ij} + \mu\left(q^*_{j,i}+q^*_{i,j}\right) + \kappa\left(q^*_{j,i} - \varepsilon_{ijk}\nu^*_k\right), \tag{2.5}$$

$$m^*_{ij} = \alpha\ \nu^*_{r,r}\ \delta_{ij} + \beta\ \nu^*_{i,j} + \gamma\ \nu^*_{j,i}, \tag{2.6}$$

where the comma denotes partial differentiation, δ_{ij} and ε_{ijk} are the Kronecker delta and the alternating tensor, respectively.
We now introduce the following non-dimensional variables

$$\begin{aligned} &\overrightarrow{q} = \frac{\mu}{GL^2}\overrightarrow{q}^{\,*}, \quad \overrightarrow{\nu} = \frac{\mu}{GL}\overrightarrow{\nu}^{\,*}, \quad y = \frac{y^*}{L}, \\ &x = \frac{x^*}{L}, \quad p = \frac{1}{GL}p^*, \end{aligned} \tag{2.7}$$

where the mean x-pressure gradient is scaled by $G = \mu\,Q/2L^3$ and the flow rate is denoted by Q.
In view of the above non-dimensional variables, the governing equations (2.1)–(2.3) reduce to

$$div\,\vec{q} = 0, \tag{2.8}$$

$$-(\mu+\kappa)\ curl\ curl\,\vec{q} + \kappa\ curl\,\vec{\nu} - \mu\,grad\ p = 0, \tag{2.9}$$

$$(\alpha+\beta+\gamma)\ grad\ div\,\vec{\nu} - \gamma\ curl\ curl\,\vec{\nu} + \kappa L^2\ curl\,\vec{q} - 2\kappa L^2\,\vec{\nu} = 0. \tag{2.10}$$

3. Micropolar cross flow

Here, we consider the flow of a micropolar fluid between two fixed corrugated walls at a distance $2L$ apart. The corrugation has the form of a sinusoidal wave of amplitude εL and wave number λ/L, where $\varepsilon\ (\ll 1)$ is the perturbation parameter and λ is the wavelength of proposed corrugations. Thus only slight corrugations are considered.

It is convenient to work with Cartesian coordinates (x, y, z) with $y = 0$ on the mid-plane between the two fixed corrugated walls. The two coordinates x and z are taken through the walls in a direction normal and parallel to the corrugations, respectively, as shown in Fig. 1. Due to the fact that the maximum or the minimum flow occurs at the phase difference 0° or 180° [33,34], we have to consider only the cases where the upper and lower corrugations are either in phase or half-period out of phase. The upper wall is assumed to be described by the equation, in non-dimensional form,

$$y = 1 + \varepsilon\sin\lambda x. \tag{3.1}$$

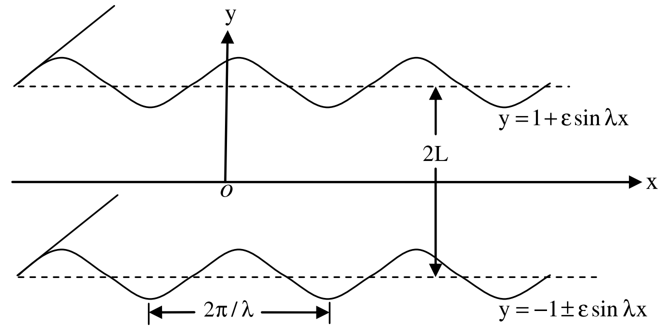

Fig. 1. Normalized section view of cross flow of a micropolar fluid through a corrugated plates.

And the lower wall is characterized by one of the following equations

$$y^{(\pm)} = -1 \pm \varepsilon\sin\lambda x, \tag{3.2}$$

where the plus (minus) sign corresponds to phase difference of 0°(180°) between the two walls. The normalized wave number is assumed to be of order unity. However, the following analysis is valid also for high wave number as far as $\varepsilon^2\lambda \ll 1$.
Suppose that the mean flow is normal to the corrugations. Then, the cross flow field is two-dimensional, and has periodic as well as non-periodic components. Therefore, the velocity and microrotation components have the forms

$$\vec{q} = (u\,(x,y)\,,\, v\,(x,y)\,,\, 0)\,, \qquad \vec{\nu} = (0, 0, \varphi\,(x,y))\,. \tag{3.3}$$

Thus, the volume flux, Q, is given by the expression $\int_{-1\pm\varepsilon\sin\lambda x}^{1+\varepsilon\sin\lambda x} u\,dy$ for all x.
Assuming that the motion is slow, then the Stokesian assumption can be applied. It is convenient to introduce the Stokes stream function ψ satisfying (2.8) which is given in Cartesian coordinates by

$$u = \frac{\partial\psi}{\partial y}, \qquad v = -\frac{\partial\psi}{\partial x}. \tag{3.4}$$

To obtain the dynamical equation satisfied by Stokes stream function, we introduce the vorticity vector

$$\vec{\omega} = \nabla\wedge\vec{q} = -\vec{k}\nabla^2\psi. \tag{3.5}$$

Therefore, one can easily get the following relations

$$\nabla\wedge\nabla\wedge\vec{\omega} = \vec{k}\nabla^4\psi, \tag{3.6}$$

$$\nabla\wedge\nabla\wedge\vec{\nu} = -\vec{k}\nabla^2\varphi, \tag{3.7}$$

where ∇^2 denotes Laplace operator and $\vec{k}$ is the unit vector perpendicular to xy-plane.
Applying the operator $\nabla\wedge$ to Eq. (2.9) and then using the relations (3.6) and (3.7) we get

$$-(\mu+\kappa)\nabla^4\psi - \kappa\,\nabla^2\varphi = 0. \tag{3.8}$$

Also, substitution of relations (3.5) and (3.7) into Eq. (2.10) with the aid of (3.3), we get

$$\left(\gamma\,\nabla^2 - 2\kappa L^2\right)\varphi - \kappa L^2\,\nabla^2\psi = 0. \tag{3.9}$$

Eliminating the microrotation φ between (3.8) and (3.9), we obtain

$$\nabla^4\left(\nabla^2 - \ell^2\right)\ \psi = 0, \tag{3.10}$$

where $\ell^2 = \frac{(2\mu+\kappa)\,\kappa\,L^2}{(\mu+\kappa)\,\gamma}$.
The non-dimensional microrotation can be then written as

$$\varphi = -\left(\tfrac{1}{2}\nabla^2\psi^{(1)} + m\ell^2\,\psi^{(2)}\right) \tag{3.11}$$