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Farfield waves created by a catamaran in shallow water

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ABSTRACT

The farfield waves created by a catamaran with identical twin hulls that advances at constant speed along a straight path in calm water of uniform finite depth are considered. The catamaran is represented as two point sources at the twin bows of the catamaran. This simple approximation of a catamaran as a 2-point wavemaker is realistic for the high-speed regime considered here. Constructive interferences between the divergent waves created by the two point sources result in an apparent wake angle, where the highest waves are found, that can be much narrower than the wake angle associated with the cusps or the asymptotes of the well-known wave patterns radiated from a single point. The farfield wave pattern of a high-speed catamaran can then differ markedly from the wave pattern obtained in the classical analysis of the waves created by a ship modeled as a 1-point wavemaker, in which interference effects are ignored. Moreover, the highest waves created by a catamaran can be much shorter than the waves at the cusps of the wave pattern, and significantly shorter than the ship length. Indeed, the wavelength of the highest waves is equal to the spacing between the twin hulls of the catamaran at high Froude numbers. Wave interferences between the twin bows of a catamaran (dominant at high Froude numbers) are compared to wave interferences between the bow and the stern of a monohull ship or a catamaran (significant at moderate Froude numbers).

1. Introduction

The farfield waves created by a catamaran with two identical hulls of length L separated by a lateral distance S that advances at constant speed V along a straight path in calm water of uniform finite depth D are considered, as illustrated in Fig. 1. The Froude numbers F_L, F_S and F_D based on the ship length L, the spacing S between the twin hulls of the catamaran, or the water depth D are defined as

$$F_L \equiv V/\sqrt{gL} \tag{1a}$$

$$F_S \equiv V/\sqrt{gS} \tag{1b}$$

$$F_D \equiv V/\sqrt{gD} \tag{1c}$$

where g denotes the acceleration of gravity. The alternative nondimensional water depths

$$d^L \equiv D/L \tag{2a}$$

$$d^S \equiv D/S \tag{2b}$$

$$d^V \equiv Dg/V^2 \equiv 1/F_D^2 \tag{2c}$$

and the identities

$$d^V \equiv d^L/F_L^2 \equiv d^S/F_S^2 \tag{2d}$$

are also used further on.

Water depth effects are negligible, i.e. the water depth is effectively infinite, if

$$d_\infty^V < d^V \quad \text{or} \quad F_D < F_D^\infty \tag{3a}$$

$$\text{with} \quad d_\infty^V \approx 3 \quad \text{and} \quad F_D^\infty \approx 0.6. \tag{3b}$$

The nondimensional water depth d^V is used hereafter instead of the equivalent water-depth Froude number F_D.

The waves created by the catamaran are observed from a Galilean frame of reference attached to the moving ship, and are represented in terms of the speed-scaled nondimensional coordinates (x, y, z) and the corresponding Fourier variables (α, β, k) defined as

$$(x, y, z, \lambda) \equiv (X, Y, Z, \Lambda)g/V^2 \tag{4a}$$

$$(\alpha, \beta, k) \equiv (A, B, K)V^2/g \tag{4b}$$

$$\text{where} \quad k \equiv \sqrt{\alpha^2 + \beta^2} \equiv 2\pi/\lambda \tag{4c}$$

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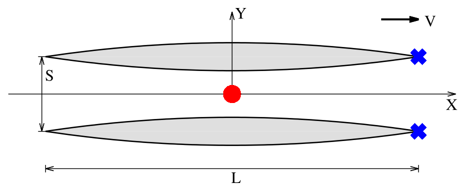

Fig. 1. Top view of a catamaran that consists of two identical demi-hulls of length L, separated by a distance S, that advances in calm water at constant speed V along the x axis. The figure also shows the 2-point wavemaker model considered here and the classical 1-point wavemaker model.

denotes the wavenumber and λ is the wavelength. The x axis is taken along the track of the ship and points toward the ship bow. The z axis is vertical and points upward, and the undisturbed free surface is chosen as the plane $z = 0$.

The waves created by the catamaran are analyzed within the classical framework of linear potential flow theory. This practical theoretical framework is realistic to analyze the farfield waves that are of primary interest here. Within that framework, the flow created by the catamaran can be represented via a distribution of sources and sinks (and possibly dipoles) over the twin hull surfaces, i.e. a surface distribution of a Green function $G \equiv L + W$ (and its gradient) where W and L denote the waves and a local flow contained in G, as well known.

Interference effects among the elementary waves radiated from every point of a ship hull surface have recently been considered by the authors in [1–4] for a monohull ship and a catamaran in deep water. These previous studies, summarized in [5], show that constructive interferences among the divergent waves created by elementary sources and sinks distributed over a ship hull-surface result in interesting effects on farfield ship waves at high Froude numbers. Specifically, the highest waves created by a fast ship are found at ray angles ψ that are significantly smaller than the classical Kelvin cusp angle. These highest-waves angles provide a simple theoretical explanation of the observations of narrow ship wakes reported in [6–11]. These observations have motivated a number of theoretical studies, including [12–22] and the authors' already mentioned studies [1–5] where a review of this literature may be found.

The numerical analysis of wave interference effects for a distribution of sources and sinks over the hull surface of a monohull ship considered in [2] shows that the elementary analysis given in [1] for a 2-point wavemaker model that consists of a point source at the bow and a point sink at the stern of the ship is a realistic model for common high speed monohull ships. This 2-point wavemaker model has been extended to the more general, and considerably more complicated, case of uniform finite water depth (shallow water) in [23].

The numerical analysis of wave interference effects for a surface distribution of sources and sinks over the twin hulls of a catamaran considered in [3] shows that the elementary analysis given in [1] for a 2-point wavemaker model that consists of two point sources at the twin bows (or two point sinks at the twin sterns) of the catamaran, as shown in Fig. 1, is a realistic model for high speed catamarans. Indeed, the 2-point wavemaker model depicted in Fig. 1 is shown in [3] to be quite accurate even for moderately high Froude numbers, specifically for Froude numbers $1 \leq F_L$. The analysis of this 2-point wavemaker model, briefly considered in [24], is expounded here.

As previously found in [23] for a 2-point wavemaker model of a monohull ship, the analysis of interferences between the divergent waves created by the pair of point sources in the 2-point wavemaker model of a catamaran depicted in Fig. 1 shows that the apparent wake angle that corresponds to the highest waves can be much narrower than the wake angle associated with the cusps or the asymptotes of the well-known wave patterns, depicted in e.g. [23], radiated from a single point. The farfield wave pattern of a high-speed catamaran can then differ markedly from the wave pattern obtained in the classical analysis of the waves created by a ship modeled as a 1-point wavemaker, in which interference effects are ignored. Moreover, the highest waves created by a catamaran can be much shorter than the waves at the cusps of the wave pattern, and significantly shorter than the ship length. The results of analysis of interference effects for a 2-point wavemaker model of a catamaran are similar to the results obtained in [23] for a 2-point wavemaker model of a monohull ship in a general sense. However, interference effects for the 2-point wavemaker model of a monohull ship previously considered in [23] and for the 2-point wavemaker model of a catamaran considered here are notably different in some respects, as also found in [1] for deep water.

2. Classical relations

Classical elementary relations, given in e.g. [23], that are required further on are now summarized. The waves created by a ship can be represented as a linear superposition of elementary waves that travel at angles $-\pi/2 < \gamma < \pi/2$ from the path of the ship (the x axis). These elementary wave solutions satisfy the Laplace equation in $-d^V < z < 0$ and the sea-floor boundary condition $\phi_z = 0$ at $z = -d^V$, and are then given by

$$e^{i(\alpha x+\beta y)} \frac{\cosh[k(z+d^V)]}{\cosh(kd^V)} \tag{5}$$

where the coordinates (x, y, z) and the Fourier variables (α, β, k) are defined by (4). The elementary wave function (5) also satisfies the linearized free-surface boundary condition

$$\phi_z + \phi_{xx} = 0 \quad \text{at } z = 0$$

if the Fourier variables (α, β, k) satisfy the dispersion relation

$$\Delta(\alpha, \beta; d^V) \equiv \alpha^2 - kt = 0 \tag{6}$$

for steady ship waves in uniform finite water depth. Here, t is defined as

$$0 \leq t \equiv \tanh(kd^V) \leq 1. \tag{7}$$

Within the first quadrant $0 \leq \alpha, 0 \leq \beta$ of the Fourier plane, the relation (6) defines the wavenumbers α and β in terms of the wavenumber k via the relations

$$\alpha = \sqrt{kt}, \qquad \beta = \sqrt{k(k-t)} \tag{8a}$$

$$\text{where} \quad k_{\min}(d^V) \leq k. \tag{8b}$$

The smallest wavenumber $k_{\min}(d^V)$ is given by

$$k_{\min}(d^V) = \begin{Bmatrix} 0 \\ \text{root of } k = t \end{Bmatrix} \quad \text{if } \begin{Bmatrix} d^V \leq 1 \\ 1 < d^V \end{Bmatrix}. \tag{9}$$

Expressions (8) and the classical farfield stationary-phase analysis yield the parametric equations

$$\begin{Bmatrix} -x_n \\ y_n \end{Bmatrix} = \frac{2n\pi\alpha/k^3}{(1-a^d)t} \begin{Bmatrix} (1-a^d)\alpha^2 + 2\beta^2 \\ (1+a^d)\alpha\beta \end{Bmatrix} \tag{10a}$$

$$\text{where} \quad 0 \leq a^d \equiv 2kd^V / \sinh(2kd^V) \leq 1 \tag{10b}$$

and $k_{\min}(d^V) \leq k$. The parametric equations (10) with $n = 1, 2, 3, \ldots$ define successive wave crests $(-x_n, y_n)$ in terms of the wavenumber k. The wave patterns defined by (10) are well known and depicted in [23] for deep water and for five finite water depths $d^V = 1.5, 1.1, 0.9, 0.5, 0.1$. The wave patterns depicted in [23]