European Journal of Mechanics B/Fluids 59 (2016) 219–228

Contents lists available at ScienceDirect

European Journal of Mechanics B/Fluids

journal homepage: www.elsevier.com/locate/ejmflu

A novel vortex scheme with instantaneous vorticity conserved boundary conditions

J.H. Pang, Z. Zong *, L. Zou, Z. Wang

State Key Laboratory of Structural Analysis for Industrial Equipment, School of Naval Architecture, Dalian University of Technology, Dalian 116024, China

HIGHLIGHTS

- A novel method to deal with the boundary vortices.
- The instantaneous circulation is conserved at each time-step.
- The fluctuating pressure coefficient can be obtained well.
- The accuracy of the surface pressure coefficient improves better.
- This scheme can be extended to the other vortex method and any bluff body.

ARTICLE INFO

Article history:
Received 13 May 2015
Received in revised form
29 December 2015
Accepted 30 December 2015
Available online 24 May 2016

Keywords:
Stream function based vortex method
Boundary vortex blobs
Instantaneous vorticity
Circle theorem
Fluctuating fluid forces

ABSTRACT

In the existing stream function based vortex methods, the solution to deal with the boundary vortex blobs is mainly eliminating the blobs that penetrate the cylinder and compensating the lost vorticity in the next time step. However, the instantaneous vorticity cannot be guaranteed to be zero, which leads to inaccuracy fluid forces. In this paper, a novel vortex scheme based on instantaneous vorticity conserved boundary conditions (IVCBC) is proposed to deal with the boundary vortices over a circular cylinder. Instead of eliminating the blobs inside, it introduces identical number of new vortex blobs outside the cylinder to counteract the strengths of the ones inside to ensure the instantaneous vorticity zero, and keep the cylinder's surface being streamline. Long time simulation of two-dimensional viscous incompressible flows is performed at Reynolds numbers 9.5×10^3, 5.5×10^4, 1×10^5 and 1.4×10^5. The results reveal that the proposed method is converged, and it significantly improves the precision of predicting the fluid forces, especially the fluctuating fluid forces and the Reynolds stress. Furthermore, this method can be extended to models of multiple circular cylinders, and any shape bluff bodies.

1. Introduction

In recent years, the vortex method based on stream function-boundary integer [1] is widely used to simulate flows around bluff bodies (Liang et al. [2], Afungchui et al. [3], Huang et al. [4], Fu et al. [5], Sun et al. [6], Chen et al. [7], Taylor et al. [8], Larsen et al. [9], Yamamoto et al. [10]). This method is very promising for its following advantages: firstly, only a small part of the flow region where vorticity appears needs to be calculated; secondly, no mesh is needed and thus it is a mesh free method; thirdly, in external flows, the vortex method can treat boundary without restricting the computational domain to a finite domain; and lastly, the turbulence model is not needed at high Reynolds numbers.

The stream function based vortex method is one version of pure Lagrangian based vortex methods. According to the existing literature, vortex methods of other pure Lagrangian based versions have been studied deeply. Mustto et al. [11,12] introduced the boundary layer theory into vortex method, and the computation precision was increased. Guedes et al. [13] attempted to improve the method of Mustto et al. [11,12] by satisfying mass conservation. Huang et al. [14,15] exploited a fast vortex core spreading method to improve the panel based vortex method. Rasmussen et al. [16] presented a novel vortex method to simulate the turbulence. However, although the stream function based vortex method is of great significance for practical application, it is paid less focus and the performance needs to be improved.

Thus, we study the stream function based vortex method, and make an effort to improve this method's calculation precision. We first started to investigate the flow over a circular cylinder, and found a very important detail that is how to deal with the vortex

* Corresponding author. Tel.: +86 41184707694; fax: +86 41184707694.
E-mail address: zongzhi_paper@163.com (Z. Zong).

http://dx.doi.org/10.1016/j.euromechflu.2015.12.010
0997-7546/

blobs penetrating into the cylinder. In the previous method, the vortex blobs that penetrate the cylinder are eliminated generally, and the lost vorticity is compensated for in the next time step. This guarantees that the sum of all the vortex strengths is equal to zero during the entire time-steps. However, the total vortex strengths in each time-step are not zero, namely the instantaneous vorticity is not conserved. This will impair the accuracy of calculating the fluid forces acting on the cylinder, which will be shown in the following content.

In this paper, a novel scheme is employed to deal with the boundary vortices. Instead of eliminating the vortex blobs inside the cylinder, we introduce identical number of new vortex blobs outside of the cylinder to counteract the strengths of the ones inside through using Circle Theorem and Image Method [17]. This ensures the instantaneous vorticity is conserved. Meanwhile, the total vorticity is also conserved from the point of the entire time-steps. To verify this method, we perform simulations of two-dimensional, incompressible, unsteady flow around a circular cylinder at different Reynolds numbers. The non-slip condition is enforced on a finite number of points on the surface of the cylinder. The convection of vortices is implemented using a two-order Adams–Bashforth scheme, and the diffusion of vortices is calculated by using random walk method. Comparison with experimental results shows the correctness and effectiveness of the proposed method. Particularly this method displays its capacity in calculating the fluctuating fluid forces and Reynolds stress.

2. The governing equations

Consider the flow is immersed in an unbounded domain with a uniform flow and free stream which is assumed incompressible, 2-D and Newtonian with constant properties. Its governing equation is Navier–Stokes equation,

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho}\nabla p + \upsilon \nabla^2 \mathbf{u}, \tag{2}$$

where $\mathbf{u}$ is the velocity vector, p the pressure, ρ the density of fluid, and υ the kinematic viscosity of the fluid. All parameters in the paper are non-dimensionalized by the amplitude of velocity $\mathbf{u}$ and radius r. In this paper $Re = 2ur/\upsilon$ is Reynolds number, Δt is time-step; t is the non-dimensionalized length of time for calculation; N is vortex number. Taking the curl on both sides of the Navier–Stokes equation, we can obtain

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + \mathbf{u} \cdot \nabla \boldsymbol{\omega} = \frac{D\boldsymbol{\omega}}{Dt} = \upsilon \nabla^2 \boldsymbol{\omega} + \boldsymbol{\omega} \cdot \nabla \mathbf{u}, \tag{3}$$

where $\boldsymbol{\omega}$ is the only non-zero component of vorticity vector (in a direction normal to the plane of the flow). For 2-D flow $\boldsymbol{\omega} \cdot \nabla \mathbf{u} = 0$, and $\boldsymbol{\omega}$ has one non-zero component ω that is

$$\frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \nabla \omega = \upsilon \nabla^2 \omega. \tag{4}$$

3. The IVCBC vortex method

The IVCBC vortex method is based on stream function, and it belongs to pure Lagrangian vortex method. Differently from the conventional stream function based vortex methods, the boundary vortices are better dealt with rather than being directly eliminated. The scheme of dealing with the boundary vortices is introduced in detail in the following part.

3.1. The basic of vortex method

The solution to Eq. (4) can be obtained by using the vortex method. This method uses an algorithm that splits the convective and diffusion operator in the following form

$$\frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \nabla \omega = 0, \tag{5}$$

and the viscous diffusion part is

$$\frac{\partial \omega}{\partial t} = \upsilon \nabla^2 \omega. \tag{6}$$

The solution to Eq. (4) is given by the Biot–Savart law [18], which is the fundamental law of the magnetic force, i.e., currents induce magnetic induction intensity. Many physical phenomena in hydrodynamics can be analogized to that in electromagnetism. Therefore, the formula of induced vortex velocity can be expressed by the Biot–Savart law.

$$\mathbf{u}(\mathbf{r}, t) = -\frac{\Gamma_r}{2\pi} k(\mathbf{r} - \mathbf{r}')\{1 - \exp[-w(r^2/\sigma^2)]\} + \mathbf{u}_\infty, \tag{7}$$

in this particular equation $\mathbf{u}_\infty$ denotes the velocity at infinity, $w = 1.25643$ is a constant. The radius of vortex core is

$$\begin{aligned} k(\mathbf{r} - \mathbf{r}') &= \frac{-(y - y')(x - x')}{|\mathbf{r} - \mathbf{r}'|} \\ \varepsilon(r) &= c\sqrt{\upsilon dt}, \end{aligned} \tag{8}$$

where $\mathbf{r}'$ is the location of vortex points, c equal to 2, and $\mathbf{r}$ the field location in the velocity field. Thus according to the Biot–Savart law, the velocity vector of each vortex can be calculated. The convection motion of each vortex generated on the body surface can be determined by integrating each vortex path equation, which can be written as

$$\begin{aligned} \Delta \mathbf{x}_c &= \mathbf{u}(\mathbf{r}(t), t)\Delta t \\ \Delta t &= \frac{2\pi k}{N}, \end{aligned} \tag{9}$$

where $\Delta \mathbf{x}_c$ is the displacement of a vortex blob resulting from convection, $\mathbf{r}(t)$ its position vector, and $\mathbf{u}$ its velocity vector, N vortex numbers on the surface of circular cylinder. The time step Δt is calculated from an estimate of the convective length and velocity scales of the flow.

The process of viscous diffusion, governed by Eq. (6), is simulated by the random walk method [19]. The form of Eq. (6) is the same with the one-dimensional heat diffusion equation, and its fundamental solution is a Green function, which is similar to the probability of a random variable. For the two-dimensional cases, the probability density function is irrelevant. Thus, the random walk of the vortex blob in the x and y directions can be obtained by solving Eq. (8) so that Δx and Δy are calculated from

$$\begin{aligned} \Delta x &= \sqrt{8Re^{-1}\Delta t \ln(1/P)\cos(2\pi Q)} \\ \Delta y &= \sqrt{8Re^{-1}\Delta t \ln(1/P)\sin(2\pi Q)}, \end{aligned} \tag{10}$$

where Re is Reynolds number, P and Q are random numbers in the interval of 0 and 1. If the positions of the vortex blobs and the time-step are given, the total displacements of vortex blob calculated by Adams–Bashforth method.

$$\begin{aligned} x_i(t + \Delta t) &= x_i(t) + [1.5u_i(t) - 0.5u_i(t - \Delta t)]\Delta t + \Delta x_i \\ y_i(t + \Delta t) &= y_i(t) + [1.5v_i(t) - 0.5v_i(t - \Delta t)]\Delta t + \Delta y_i. \end{aligned} \tag{11}$$